$T^\star$-Lite: A Fast Time-Risk Optimal Motion Planning Algorithm for Multi-Speed Autonomous Vehicles

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Autonomous Vehicle Path Planning Applications

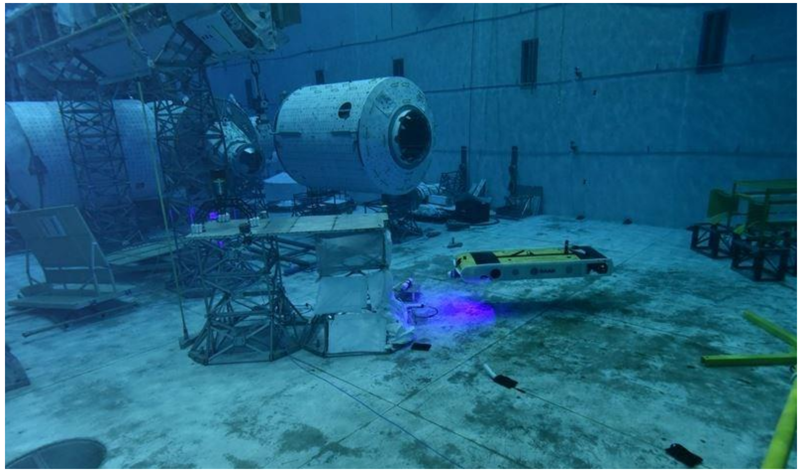
Mine Countermeasures

“Mine Countermeasures”, General Dynamics, n.p.n.d., May 16, 2011

Search & Rescue

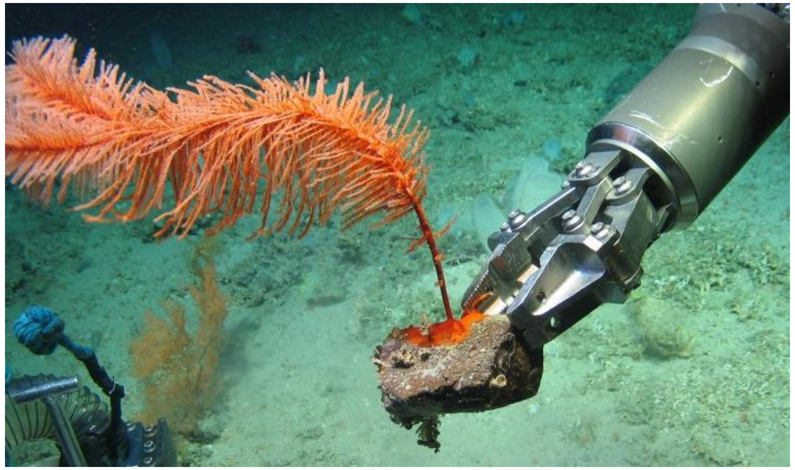
US Coast Guard Office of Search and Rescue (CG-SAR)

Oil & Gas Industry



“Saab Sabertooth AUV/ROV for Oil & Gas Inspection”, Saab, Jan 26,2018

Wildlife Habitat Monitoring



Deep-sea coral specimen collection. Plymouth University et al. 2018

Introduction

Objective: Develop a computationally efficient time-risk optimal motion planner for variable-speed autonomous vehicles in obstacle-rich environments.

Existing Approaches

$T^\star$ for Time-Risk Optimal Motion Planning

- The **$T^\star$ algorithm** [1] is the only motion planner that considers multi-speed vehicles and jointly optimizes time and risk.
- **Limitation**: Grid-based approach is computationally expensive

Sample-based Methods for Rapid Motion Planning

- RRT* and PRM* [2] quickly generate asymptotically-optimal shortest paths as number of samples increase
- **Limitation**: Restricted to single-speed vehicles & no risk considered

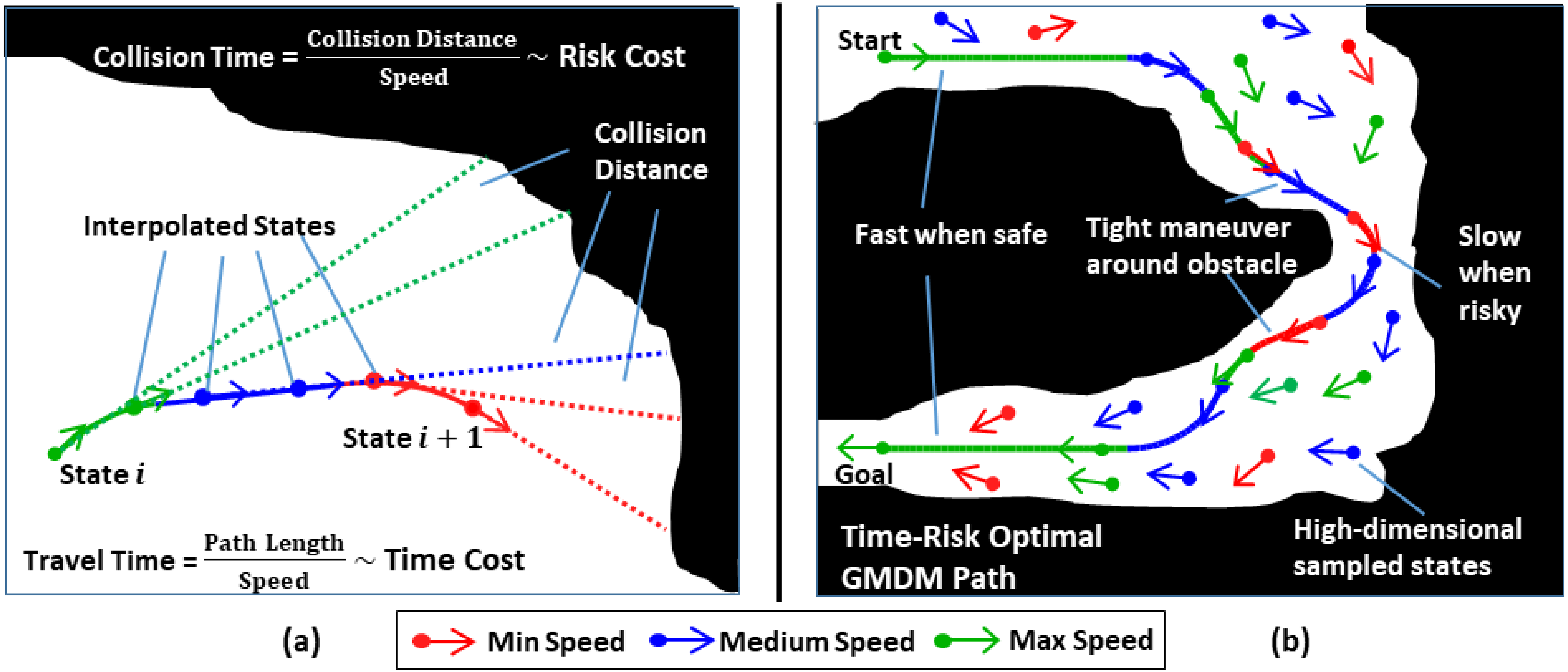
Kinodynamic Motion Models

- Dubins [3] provides shortest paths for single velocity vehicles
  - **Limitation**: Does not consider multi-speed vehicles
- Wolek [4] provides time-optimal paths for multi-speed vehicles
  - **Limitation**: Requires nonlinear solvers
- The recently developed ***Generalized Multi-speed Dubins Motion Model (GMDM)*** [5] overcomes the above limitations:
  - Better maneuvering by controlling the turning radius
  - Speed selected based on obstacle distance to mitigate risk
  - Allows for real-time computation

Features and Contributions of $T^\star$-Lite

- Enables fast time-risk optimal motion planning for variable-speed vehicles by:
  - Porting the novel time-risk cost function from $T^\star$ into a fast and asymptotically-optimal sample-based motion planner
  - Generating samples from a four-dimensional configuration space considering position, heading, and speed.
  - Utilizing the GMDM to produce the optimal time-risk trajectories connecting sampled states
- Algorithm is computationally efficient while providing reasonable solution quality

T*-Lite Overview



(a) Overview of the computation of the time and risk costs in the joint optimization problem.

(b) Example of the high-dimensional sampled vehicle states and the time-risk optimal trajectory produced by the Generalized Multi-speed Dubins Motion Model.

Problem Formulation

Autonomous Vehicle Description & Search Area

Autonomous Vehicle Description

☐ $(x, y, \theta) \in SE(2)$ is the vehicle and position heading

☐ Taking speed $v(t)$ and turning rate $u(t)$ be the inputs, the equations of motion are:

$$\begin{cases} \dot{x}(t) = v(t) \cdot \cos\theta(t) \\ \dot{y}(t) = v(t) \cdot \sin\theta(t) \\ \dot{\theta}(t) = u(t) \end{cases}$$

Bounded Turning Rate
$u(t) \in [-u_{\max}, u_{max}]$ rad/s, $u_{\max} \in \mathbb{R}^+$ is the max turning rate, and “-/+” indicates a *right/left* turn

Variable Speed
$v(t) \in [v_{min}, v_{max}]$ m/s

Curvature
$\kappa(t) = \frac{u(t)}{v(t)}, \quad 0 \leq |\kappa(t)| \leq \frac{u_{max}}{v_{min}}$

Note: curvature is the inverse of turning radius: $r(t) = \frac{1}{\kappa(t)}$

Search Area: $A \in \mathbb{R}^2$

Define vehicle state as $\mathbf{p} = (x, y, \theta, v)$

☐ $(x, y) \in A_{free}$ ☐ $\theta \in [0,2\pi)$ ☐ $v \in [v_{min}, v_{max}]$

Problem Formulation

Time-Risk Cost Function

Admissible Control: Let Γ denote the set of collision-free paths between the start state $\mathbf{p}_{start}$ and goal state $\mathbf{p}_{goal}$. For each path $\gamma \in \Gamma$, the control $\boldsymbol{c}(s) = (\boldsymbol{\kappa}, v)$ at any point s on path γ, belongs to:

$$\boxed{\Omega = \left\{(\kappa, v) \colon v_{\min} \leq v \leq v_{max}, |\kappa| \leq \frac{u_{\max}}{v}\right\}}$$

Cost of a Path: Let $R(s)$ denote the risk cost at point s on path γ. Then the total cost is written as:

$$\boxed{J(\gamma) = \int_{\gamma} \underbrace{\boxed{R(s)}}_{\text{risk cost}} \cdot \underbrace{\boxed{\frac{1}{v(s)}}}_{\text{time cost}} ds}$$

Objective: Find the optimal control $c^{\star} \in \Omega$, which generates the collision-free path $\gamma^{\star}$, such that: $J(\gamma^{\star}) \leq J(\gamma), \forall \gamma \in \Gamma$ in a computationally efficient manner

T*-Lite Algorithm

Approximate Time-Risk Cost Function

Approximate Piecewise Path Cost Function: Assume a constant risk along path $\gamma_{i,i+1}$. Thus:

$$J(\gamma_{i,i+1}) = \boxed{R(\gamma_{i,i+1})} \cdot \boxed{\int_{\gamma_{i,i+1}} \frac{1}{v(s)} ds}$$

risk cost time cost

Risk Cost $R(\gamma_{i,i+1})$: For each evenly interpolated state $\hat{p}_\ell$ along $\gamma_{i,i+1}$:

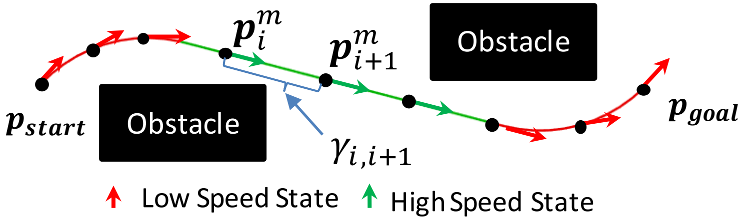
1. Compute ***collision time*** $t_\ell = \frac{d_l}{v_\ell}$
2. Given safety threshold $t^\star$, compute sample risk:

$$risk(\hat{p}_\ell) = \begin{cases} 1 + \log\left(\frac{t^\star}{t_\ell}\right) & \text{if } t_\ell < t^\star \\ 1 & if\ t_\ell \geq t^\star \end{cases}$$

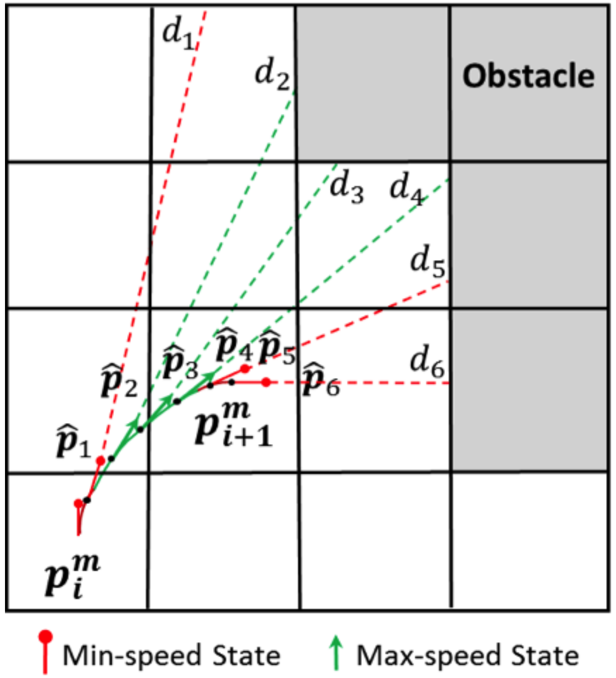
Finally, the piecewise risk is computed as:

$$R(\gamma_{i,i+1}) = \max_{\ell \in \{1,\ldots,M\}} (risk(\hat{p}_\ell))^k$$

☐ $k > 0$ is the user-defined *risk weight*



An example of the interpolated state sequence P^m composed of states $\boldsymbol{p}_i^m = (x_i, y_i, \theta_i, v_i)$



Sample states $\hat{\boldsymbol{p}}_\ell$ along collision-free path $\gamma_{i,i+1}$ and corresponding collision distances d_ℓ

T*-Lite Algorithm

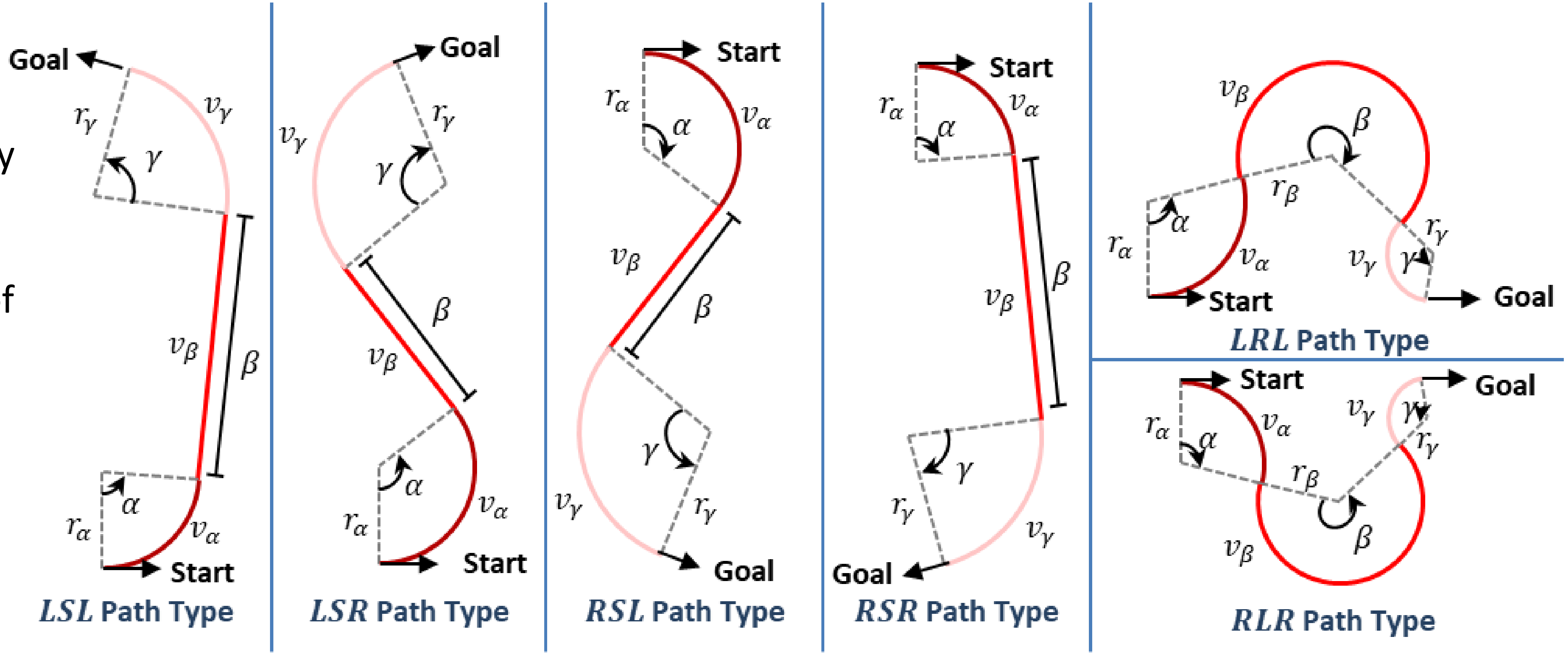
Kinodynamic Motion Model

The **Generalized Multi-speed Dubins Motion Model (GMDM)** is a fundamental improvement of the Dubins model that enables the selection of any speed for any of the three segments of a Dubins path ($L \equiv$ left turn, $S \equiv$ straight, or $R \equiv$ right turn).

- Used to generate a set of candidate trajectories that connect any two states $\mathbf{p}_i$ to $\mathbf{p}_{i+1}$

Main Features:

- Provides path planners the flexibility to select the appropriate speed dynamically based on the perceived risk
- Selection of both turning rate and speed enables selection of appropriate turning radius to smoothly maneuver around obstacles based on their shapes and sizes
- Synthesis is similar to Dubins, thus providing simple closed-form solutions for real-time computation.



LSL Path Type

LSR Path Type

RSL Path Type

RSR Path Type

LRL Path Type

RLR Path Type

Visualization of the Generalized Multi-speed Dubins Motion model for each of the six path types.

T*-Lite Algorithm

RRT* Motion Planner

T*-Lite utilizes the asymptotically-optimal sample-based RRT* framework, which has six core functions:

- ☐ *Nearest neighbor*
- ☐ *Near-by vertices*
- ☐ *Collision check*

no changes from RRT*

- ☐ *Sampling*
- ☐ *Distance*
- ☐ *Local steering*

updated in T*-Lite

Summary of RRT*

- ☐ Sampled states are randomly generated in the obstacle-free space
- ☐ A search tree of minimum-cost collision-free paths that connect states to the start node is created
- ☐ As new states are added, connections between existing states are updated if connections to the new states are faster.

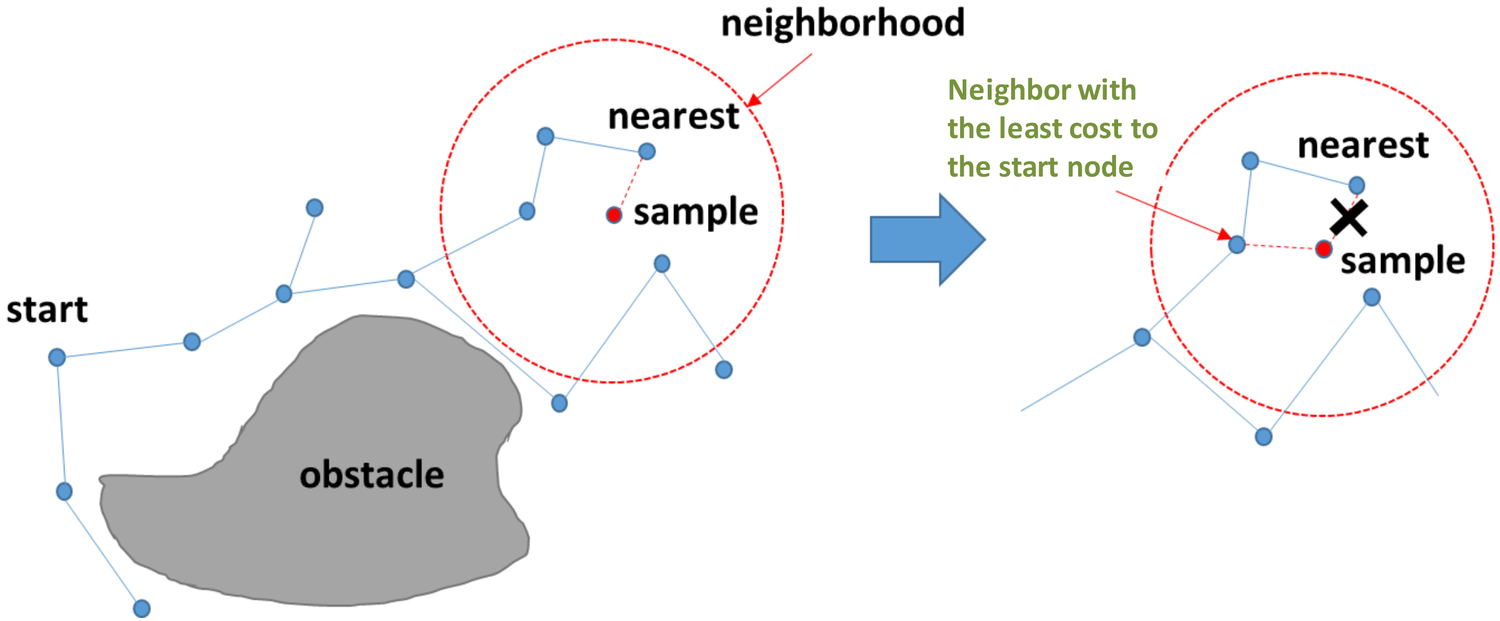


Figure: Illustration of RRT* and the iterative search-tree update for a point vehicle.
Note: connections between nodes in T*-Lite are subject to curvature constraints.

T*-Lite Algorithm

Core Functions

T*-Lite is based on the asymptotically-optimal sample-based RRT* framework, which has six core functions:

☐ *Nearest neighbor*
☐ *Near-by vertices*
☐ *Collision check*

no changes from RRT*

☐ *Sampling*
☐ *Distance*
☐ *Local steering*

updated in T*-Lite

Sampling Function*:* generates randomly sampled collision-free states states $\mathbf{p} = (x, y, \theta, v) \in \mathbf{P}$ in the obstacle-free space A_{free}.

Distance Function: Let $dist: \mathbf{P} \times \mathbf{P} \rightarrow \mathbb{R}^2$ be a function that returns the cost of the time-risk optimal trajectory $\gamma_{i,i+1}^*$ between two states $\mathbf{p}_i, \mathbf{p}_{i+1} \in \mathbf{P}$ such that $dist(\mathbf{p}_i, \mathbf{p}_{i+1}) = J(\gamma_{i,i+1}^*)$.

Local Steering Function*:* Given two states $\mathbf{p}_i, \mathbf{p}_{i+1} \in \mathbf{P}$, the *steer* function produces the optimal collision-free trajectory $\gamma_{i,i+1}^*$ connecting $\mathbf{p}_i$ to $\mathbf{p}_{i+1}$ such that $J(steer(\mathbf{p}_i, \mathbf{p}_{i+1})) = dist(\mathbf{p}_i, \mathbf{p}_{i+1})$. Producing the optimal trajectory requires:

☐ The **approximate optimization function from T*** to evaluate the time-risk costs of the created candidate trajectories
☐ A **kinodynamic motion model** to create a sufficient set of candidate trajectories connecting two states

Simulation Setup

Autonomous Vehicle and T*-Lite Parameters:

- ☐ $(v_{\min}, v_{\max}) = (0.5{,}1.0)\ m/s$
- ☐ $u_{\max} = 0.5\ rad/s$
- ☐ Safety threshold $t^\star = 6\ s$
- ☐ Risk weight $k\ = 2$
- ☐ Num. of interpolated states $M\ = 4$
- ☐ Search tree max size: 3000 sampled states
- ☐ Num. of nearest neighbors: 100
- ☐ Max. connection distance: $3m$
- ☐ Scenario Size: $30m \times 30m$

Motion models used in T*-Lite:

- ☐ Max-speed Dubins motion model
- ☐ Generalized Multi-speed Dubins Motion model

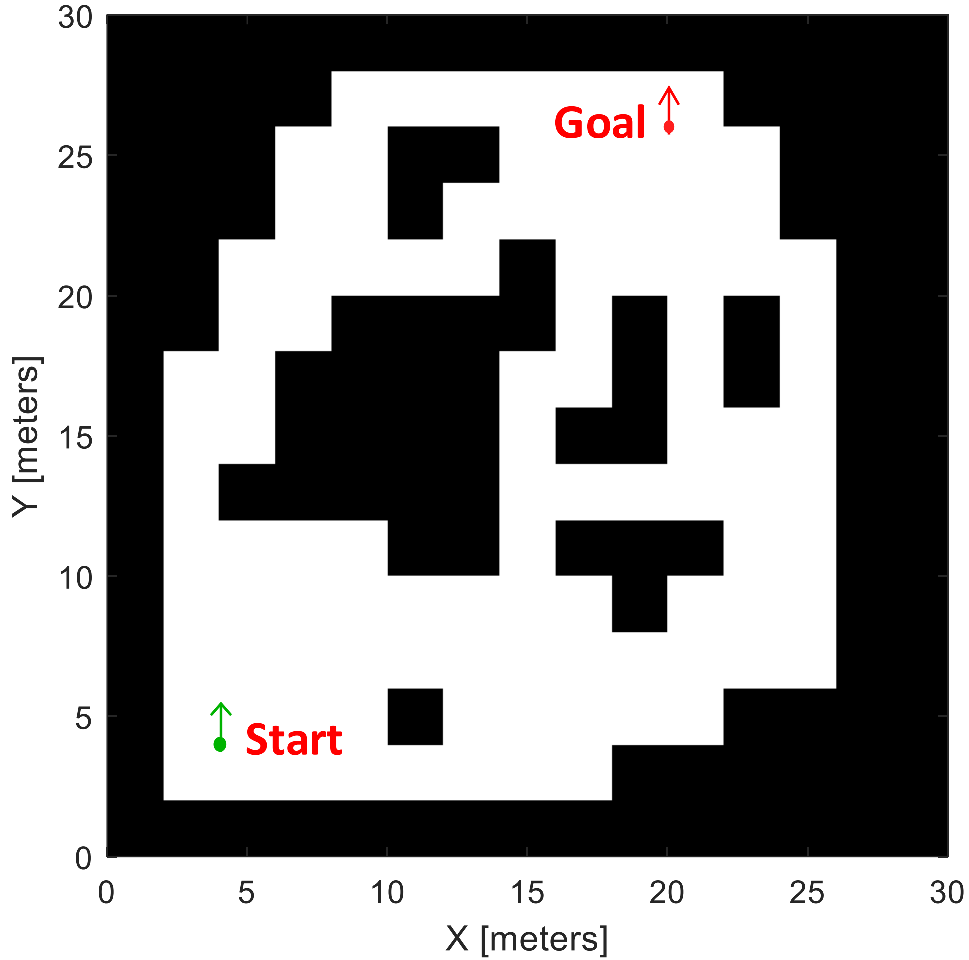
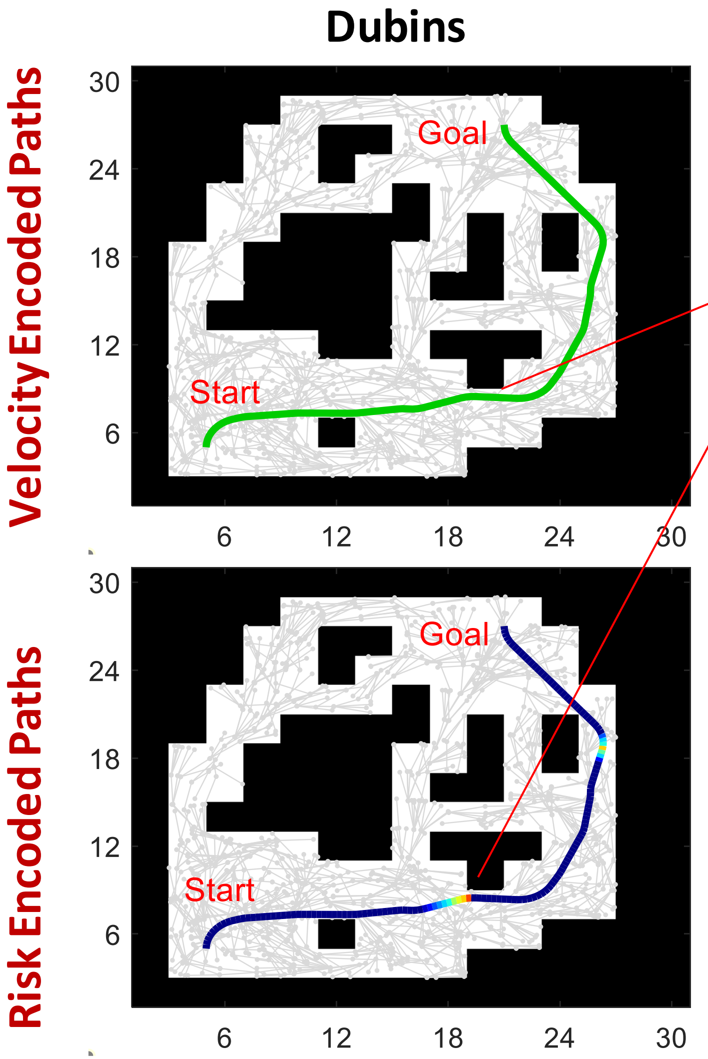


Figure: Scenario used in simulation

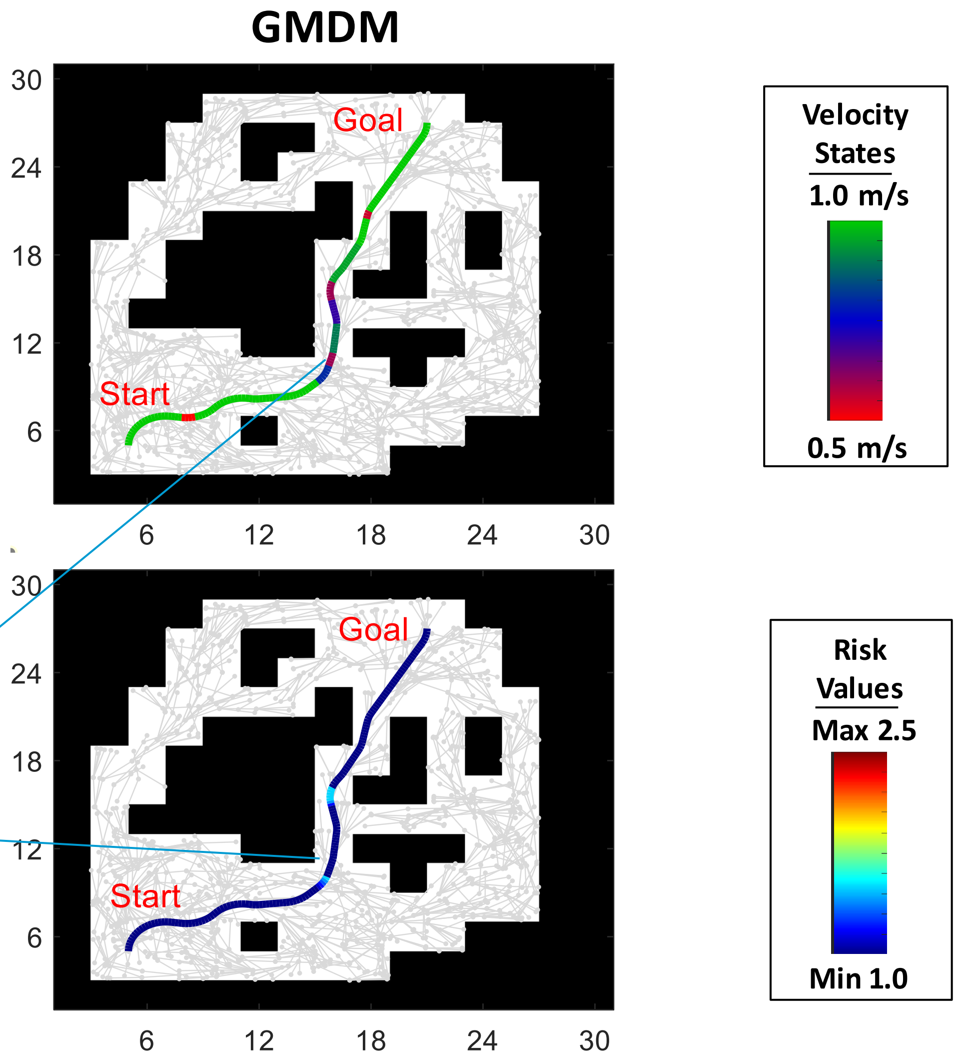
Results



Travel Time: 44.56 s **Max Risk:** 2.13
Time-Risk Total: 60.47 **CPU Time:** 1.34 s

Single speed reduces CPU requirements but produces longer and riskier paths

Variable speeds enhance maneuverability and reduce risk to provide quicker and safer paths



Travel Time: 39.23 s **Max Risk**: 1.69
Time-Risk Total: 45.78 **CPU Time:** 8.65 s

Conclusions & Future Work

Conclusions

- Developed $\mathrm{T}^\star$-Lite for rapid time-risk optimal motion planning for variable-speed autonomous vehicles. Achieved by:
  - Porting the novel time-risk cost function from $\mathrm{T}^\star$ into the RRT* framework
  - Generating high-dimensional samples that considers vehicle position, heading, and speed.
  - Utilizing the Generalized Multi-speed Dubins Motion model to provide near-optimal trajectories in a computationally efficient manner
- Provides fast, safe, and flexible maneuvers in obstacle-rich environments
- Suitable for on-demand real-time motion planning

Future Work

- In-depth analysis of the $\mathrm{T}^\star$-Lite framework in other asymptotically optimal sample-based frameworks.
- Direct comparisons against the grid-based $\mathrm{T}^\star$ in terms of both solution quality and CPU time.
- Develop smart high-dimensional sampling methods for multi-speed vehicles to further enhance solution quality and reduce computation time.
- Extend to multi-agent resilient systems.

References

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Thank You!