

ϵ^* : An Online Coverage Path Planning Algorithm

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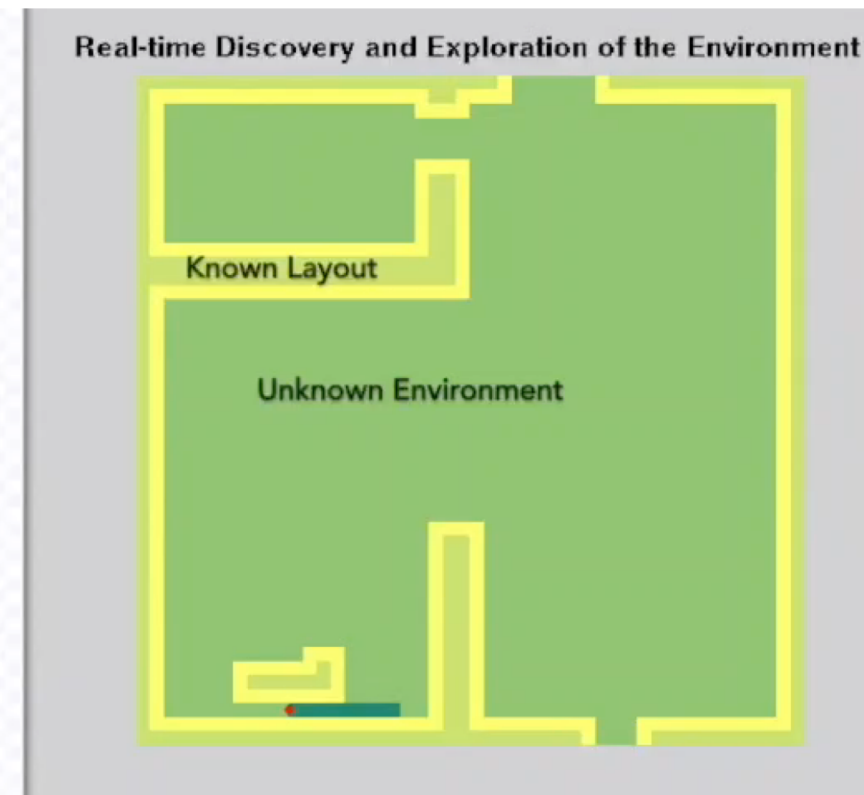
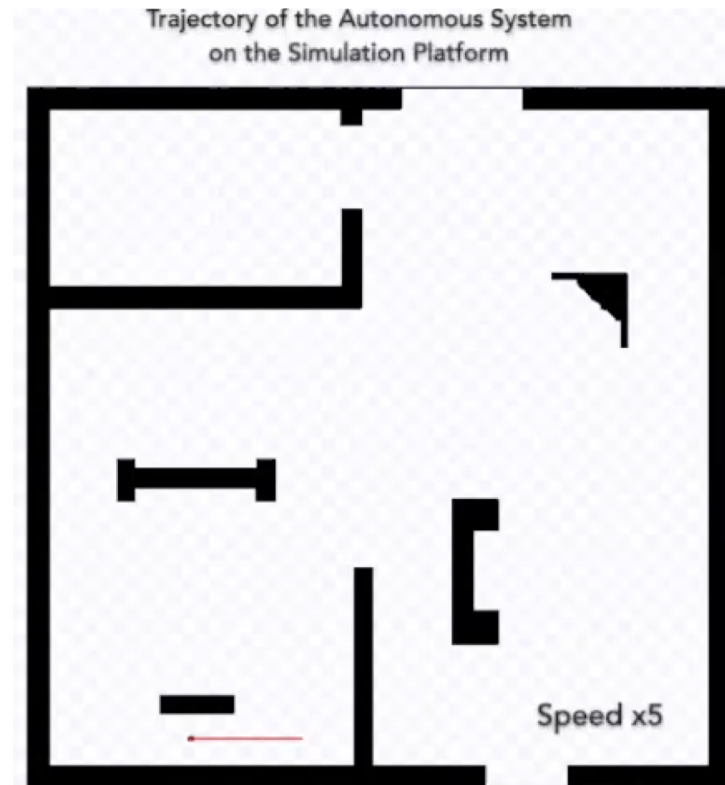
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❖ **Objective:** Develop an online coverage path planning algorithm for an autonomous vehicle in unknown environment

❖ **Challenges:**

- Online detection and avoidance of unknown obstacles
- Generate back-and-forth path with minimized turns and overlappings
- Must guarantee complete coverage and prevent any local extremum
- Low computational complexity for real-world applications



Unexplored
 Explored
 Obstacle
 Forbidden

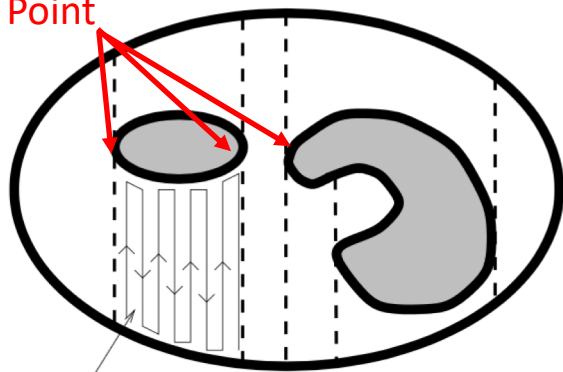
❖ Existing Approaches:

- Learning Real-time A* (LRTA*) ← *Strong path overlappings*
- Spanning-tree Coverage
- Backtracking Spiral Algorithm } *Generate spiral path with too many turns*
- Brick-and-Mortar Algorithm
- Cellular Decomposition (*back-and-forth* path)
 - Rely on detection of critical points (detection and pairing of IN & OUT critical points are difficult in complex environment)
 - Require cycle algorithm which leads to overlappings
 - Cannot work in rectilinear environment

❖ Features and Novel Contributions of the ϵ^* Algorithm:

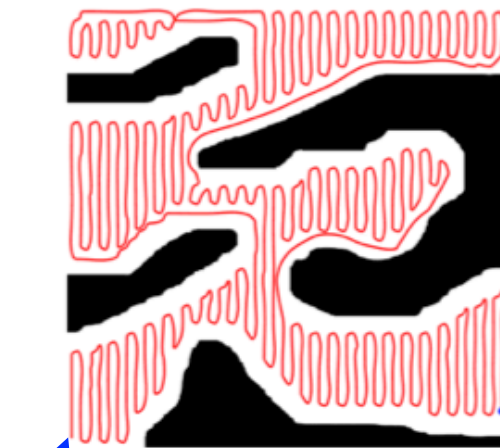
- Produces the desired *back-and-forth* path
- **Does not need critical point detection on obstacles**
- Guarantees complete coverage and prevents the local extrema problem using hierarchical potential surfaces (called MAPS)
- Capable of adapting sweep direction in known sub-regions to further reduce the number of turns
- Computationally efficient for real-time applications

Critical Point

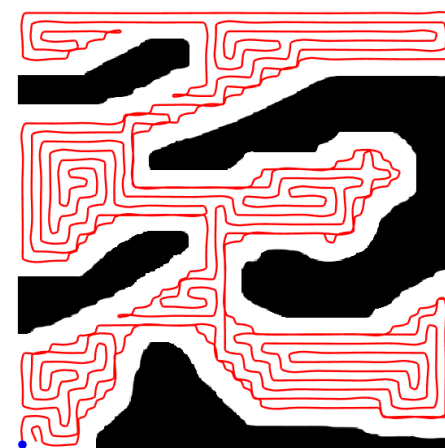


Coverage Path in a Cell

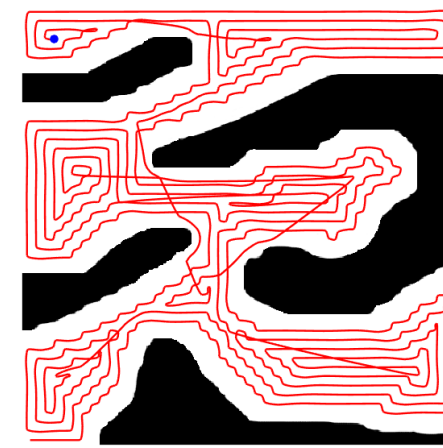
(a) Cellular Decomposition based Method^[1]



(b) ϵ^* Algorithm
Number of Turns: 291



(c) Spanning Tree Coverage
Number of Turns: 348



(d) Backtracking Spiral Algorithm
Number of Turns: 407

[1] E. U. Acar, H. Choset, A. A. Rizzi, P. N. Atkar and D. Hull, 2002. Morse decompositions for coverage tasks. The international journal of robotics research, 21(4), pp.331-344.

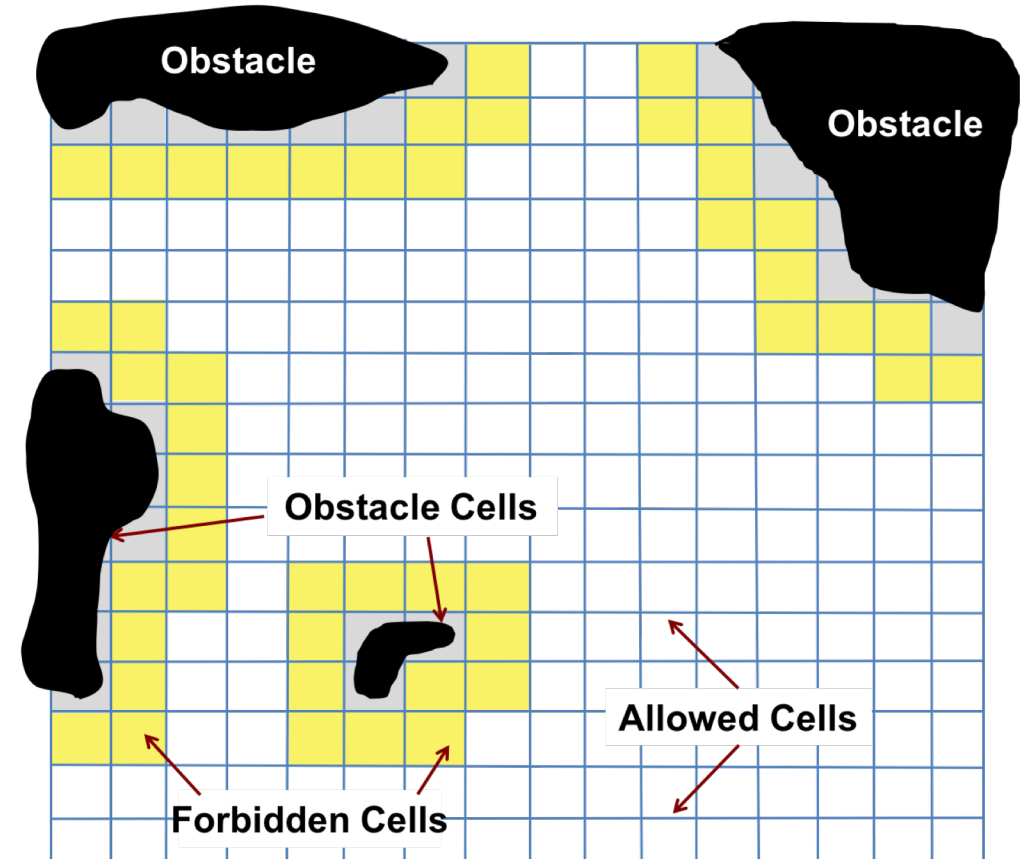
- ❖ Let $\mathcal{R} \subset \mathbb{R}^2$ be the estimated area that includes the desired area to cover.

Tiling. The set $T = \{\tau_\alpha \subset \mathbb{R}^2: \alpha = 1 \dots |T|\}$ is called a tiling of \mathcal{R} if its elements:

- i) have mutually exclusive interiors, i.e., $\tau_\alpha^o \cap \tau_\beta^o = \emptyset, \forall \alpha \neq \beta$, where $\alpha, \beta \in \{1 \dots |T|\}$.
- ii) form a minimal cover, i.e., $\mathcal{R} \subseteq \bigcup_{\alpha=1}^{|T|} \tau_\alpha$, while removal of any tile destroys the covering property.

ϵ Cell: Each element $\tau_\alpha, \forall \alpha \in \{1, \dots |T|\}$, is called an ϵ -cell.

- ❖ The tiling T is partitioned into three subsets:
 - **Obstacle cells** (T^o): they are detected online.
 - **Forbidden cells** (T^f): create buffer around obstacles
 - **Allowed cells** (T^a): these are the target cells to cover



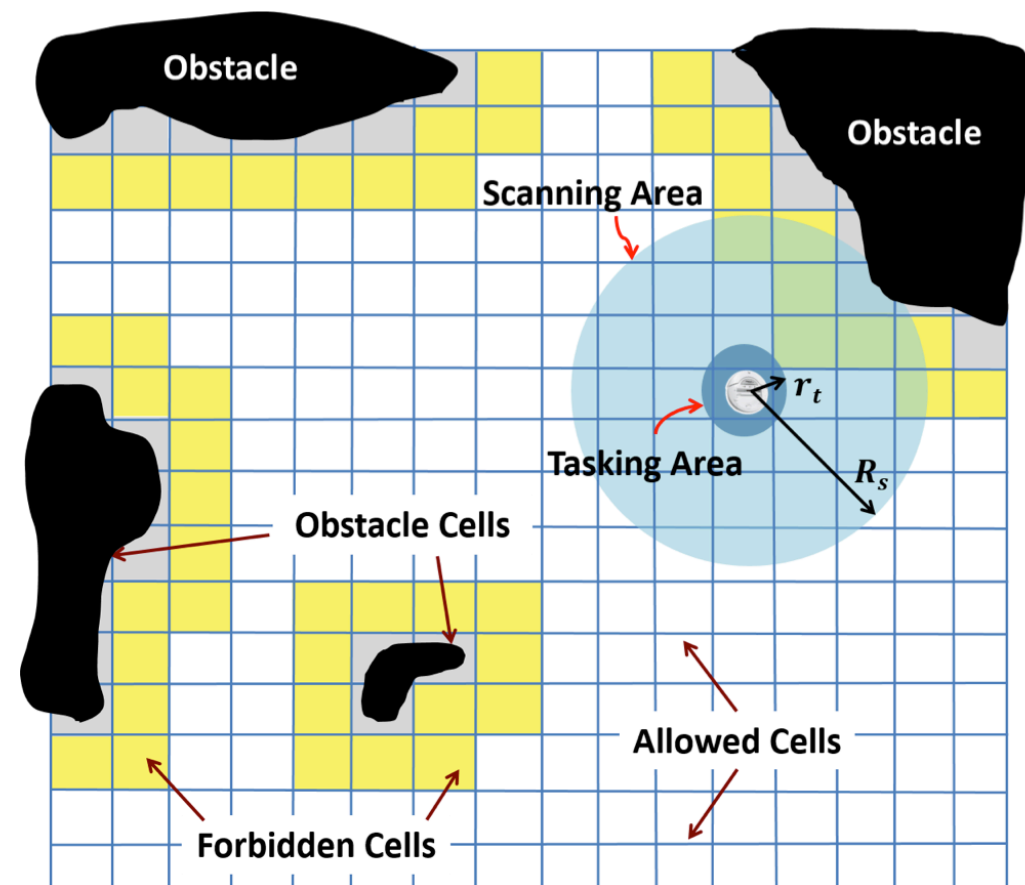
Tiling of the Search Area

❖ **The autonomous vehicle is equipped with:**

1. Localization System
 - Provides vehicle location (e.g., GPS), and heading (e.g., Compass)
2. Range Detector with Sensing Radius R_s
 - Allows the vehicle to detect obstacles in the local neighborhood (e.g., laser)
3. Tasking Sensor with Radius r_t
 - Allows the vehicle to carry out certain tasks (e.g., cleaning, target detection, crops cutting) while it operates in the field

ϵ -Coverage Let $\mathcal{R}(T^a)$ denote the total area of the allowed cells. Let $\tau(k) \in T^a$ be the ϵ -cell visited by the autonomous vehicle at time k and explored by its tasking sensor. Then, \mathcal{R} is said to achieve ϵ -coverage, if $\exists K \in \mathbb{Z}^+$, such that the sequence $\{\tau(k), k = 1, \dots, K\}$ covers $\mathcal{R}(T^a)$, i.e.,

$$\mathcal{R}(T^a) \subseteq \bigcup_{k=1}^K \tau(k)$$

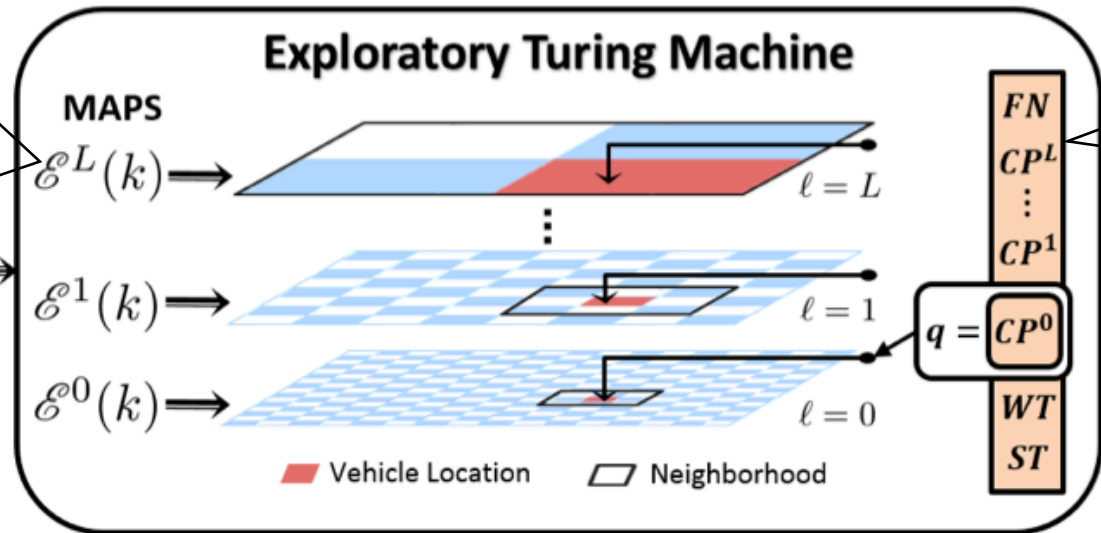


The autonomous vehicle and the tiling

The Supervisory Controller: Exploratory Turing Machine (ETM)

Multi-scale Adaptive Potential Surfaces (MAPS)

- \mathcal{E}^0 : time-varying potential surface at the finest level
- $\mathcal{E}^\ell, 1 \leq \ell \leq L$: time-varying potential surfaces at higher levels

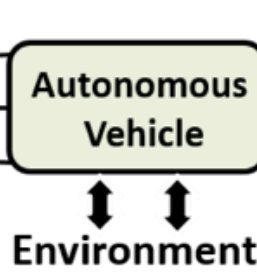


Machine States:

- $ST \equiv$ Start
- $CP \equiv$ Compute
- $WT \equiv$ Waiting
- $FN \equiv$ Finished

Input Vector i_p

- $\lambda \in \{1, \dots |T|\}$: index of current cell
- $ol \subset \{1, \dots |T|\}$, indices of obstacle cells
- $ts \in \{cm, ic\}$, tasking status, where $cm \equiv$ Complete, or $ic \equiv$ Incomplete.



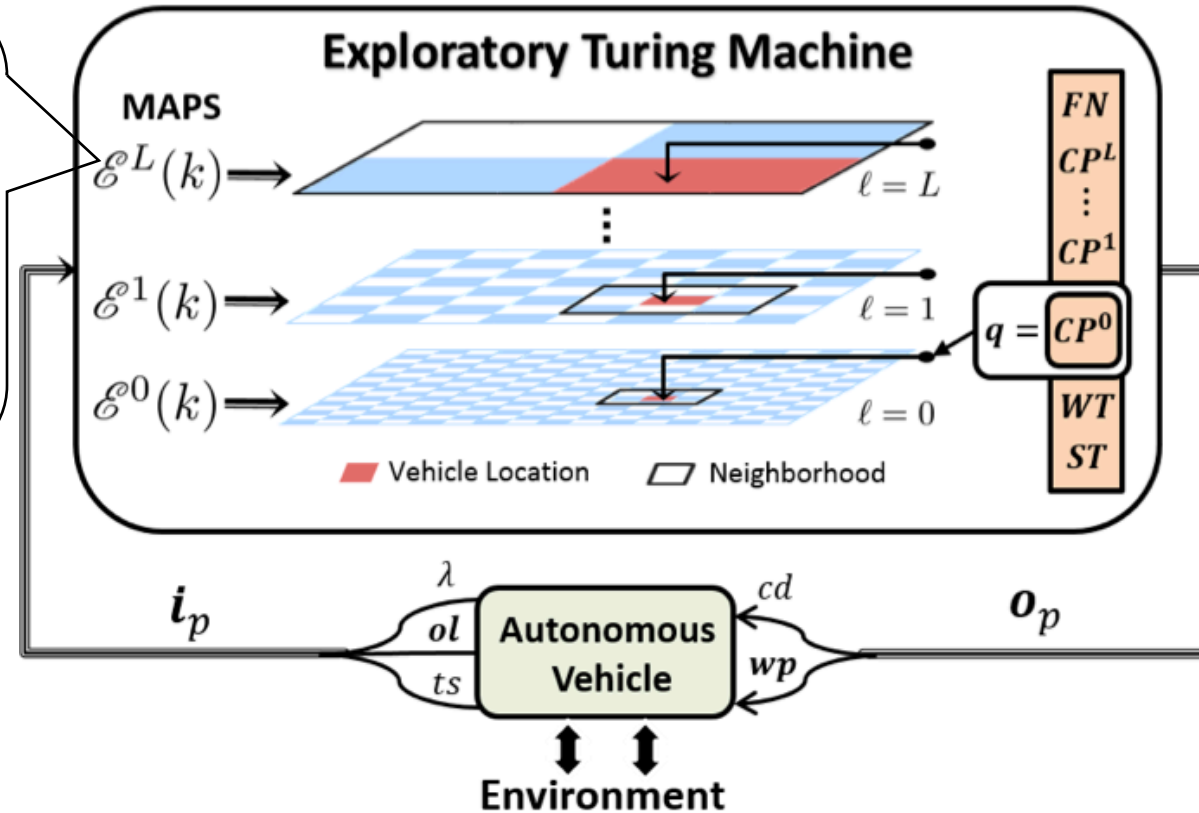
Output Vector o_p

- $cd \in \{mv, tk, id, sp\}$: command to the vehicle, where: $mv \equiv$ Move, $tk \equiv$ Task, $id \equiv$ Idle, and $sp \equiv$ Stop.
- $wp \subset \{1, \dots |T|\}$, candidate set of new waypoints for the vehicle

The Supervisory Controller: Exploratory Turing Machine (ETM)

Multi-scale Adaptive Potential Surfaces (MAPS)

- \mathcal{E}^0 : time-varying potential surface at the finest level
- $\mathcal{E}^\ell, 1 \leq \ell \leq L$: time-varying potential surfaces at higher levels



Dynamically Constructed Multi-scale Potential Surfaces (MAPS)

❖ Level 0 of MAPS

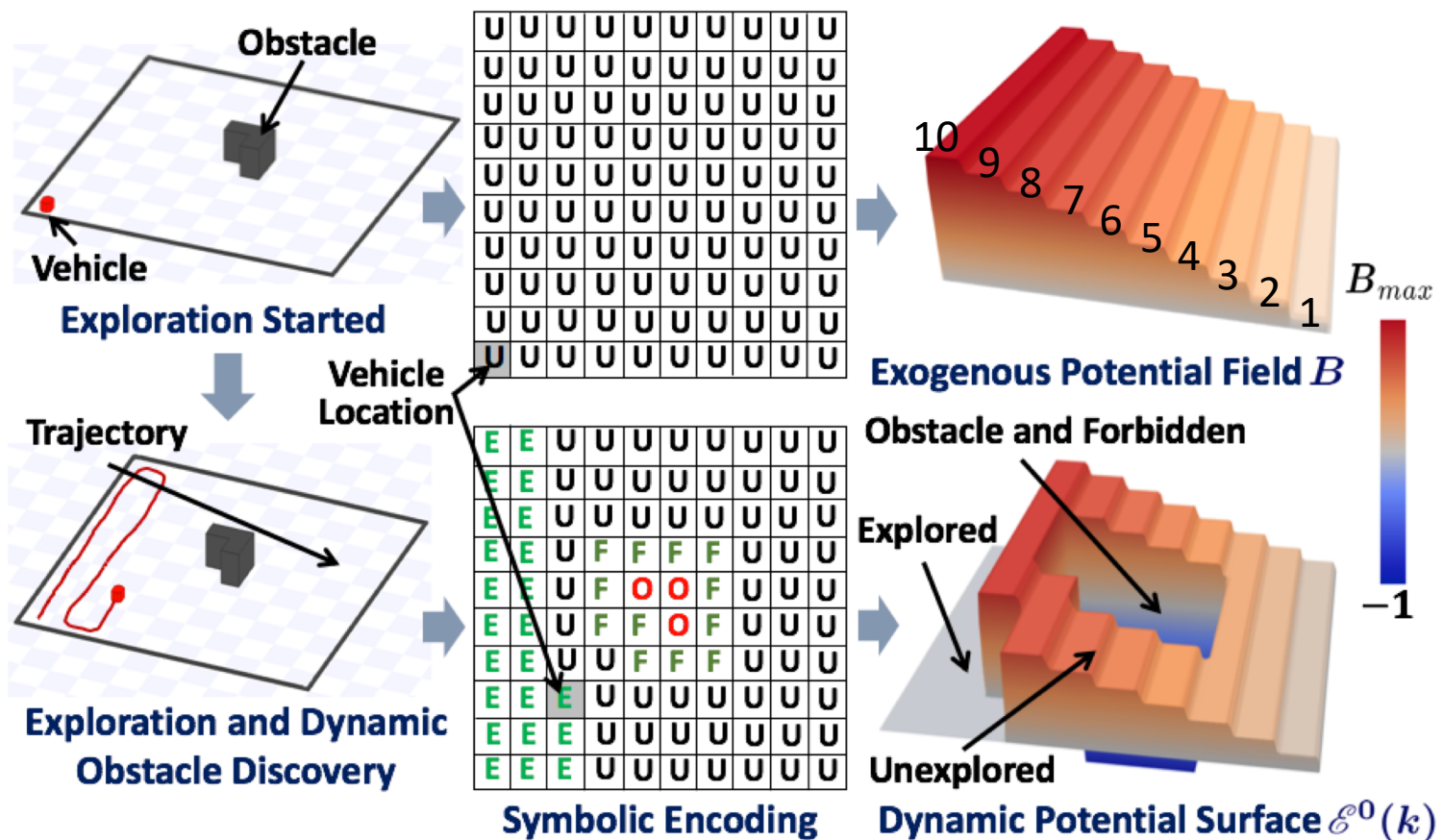
▪ **Symbolic Encoding:** each ϵ -cell at level 0, τ_{α^0} , is assigned with a symbolic state s_{α^0} , from below:

- O : Obstacle
 - F : Forbidden
 - E : Explored
 - U : Unexplored
- } Allowed cells

▪ **Potential Surface \mathcal{E}^0 :**

$$\mathcal{E}_{\alpha^0}(k) = \begin{cases} -1 & \text{if } s_{\alpha^0} = O \text{ or } F \\ 0 & \text{if } s_{\alpha^0} = E \\ B_{\alpha^0} & \text{if } s_{\alpha^0} = U \end{cases}$$

where $B = \{B_{\alpha^0} \in \{1, \dots, B_{\max}\}, \alpha^0 = 1, \dots, |T^0|\}$ is a time-invariant exogenous potential field. It is designed *offline* to have plateaus of equipotential surfaces along each column of the tiling.



Dynamically Constructed Multi-scale Potential Surfaces (MAPS)

Note: Higher levels of MAPS are used to prevent the local extrema problem.

Local Extrema: no unexplored cells are available in the local neighborhood on Level 0.

❖ Levels $\ell = 1, 2, \dots, L$ of MAPS

- **Potential Surfaces** \mathcal{E}^ℓ , $\ell = 1, \dots, L$, are constructed by assigning τ_{α^ℓ} the *average* potential generated by all the *unexplored* ϵ -cells within τ_{α^ℓ} , such that

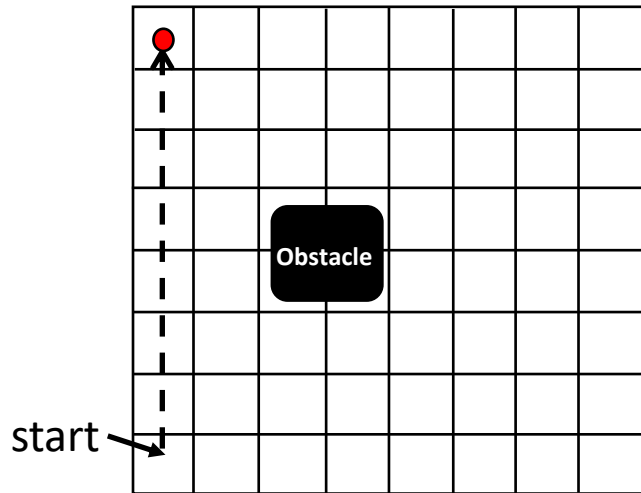
$$\mathcal{E}_{\alpha^\ell}(k) = p_{\alpha^\ell}^U(k) \cdot \bar{B}_{\alpha^\ell}$$

where \bar{B}_{α^ℓ} is the mean exogenous potential of τ_{α^ℓ} , and $p_{\alpha^\ell}^U(k)$ is the probability of *unexplored* ϵ -cells in τ_{α^ℓ} .

An Illustrative Example: Updates of MAPS at Level 0

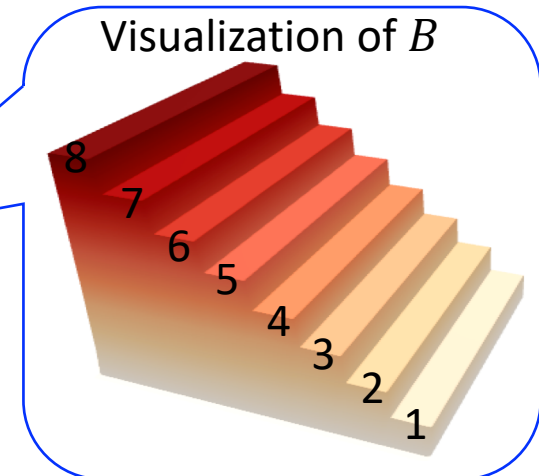
❖ An Illustrative Example

Vehicle trajectory and obstacle layout



Potentials B at Level 0

8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1



Symbolic encodings at Level 0

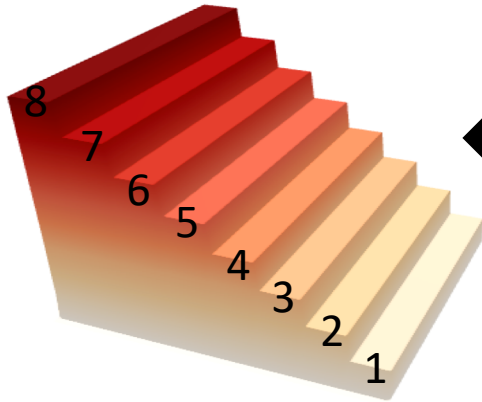
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U
E	F	F	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	F	F	U	U	U	U
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U

MAPS update on Level 0

0	7	6	5	4	3	2	1
0	7	6	5	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	-1	-1	-1	4	3	2	1
0	7	6	5	4	3	2	1
0	7	6	5	4	3	2	1

An Illustrative Example: Updates of MAPS at Level 1

Visualization of B

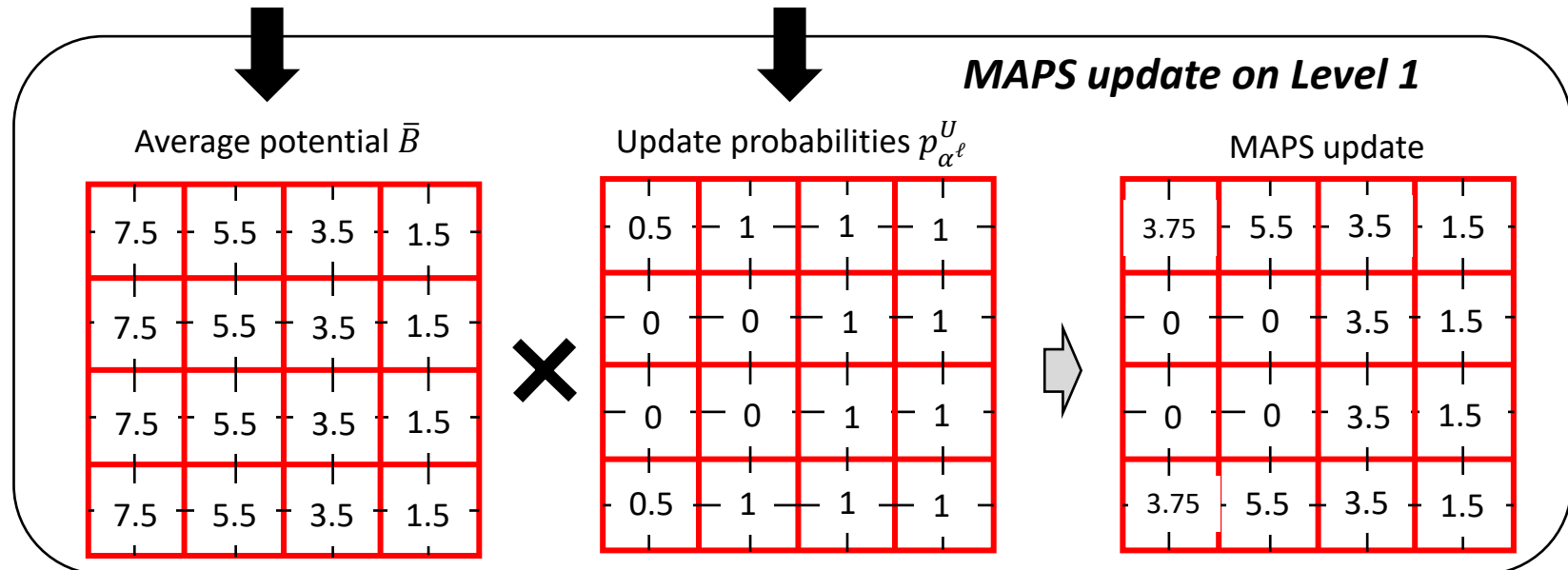


Potentials B at Level 0

8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1

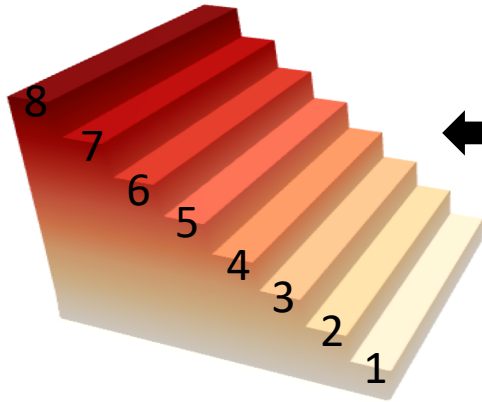
Symbolic encodings at Level 0

E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U
E	F	F	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	F	F	U	U	U	U
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U



An Illustrative Example: Updates of MAPS at Level 2

Visualization of B

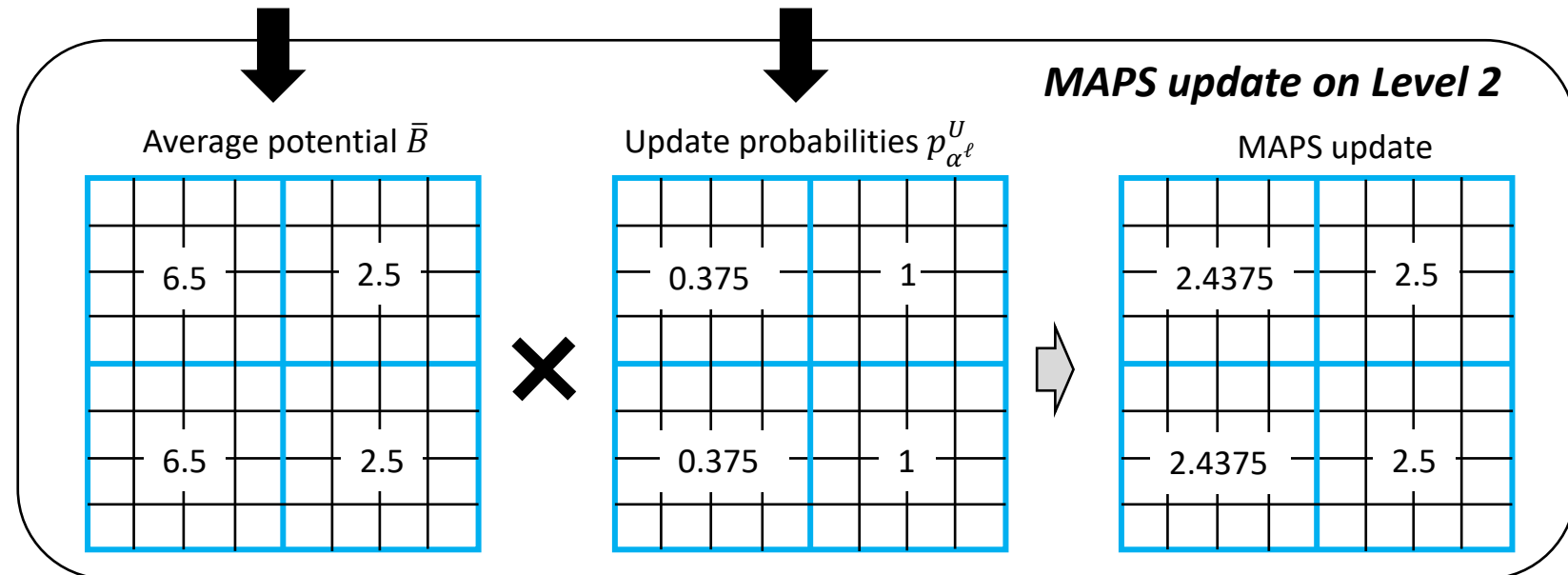


Potentials B at Level 0

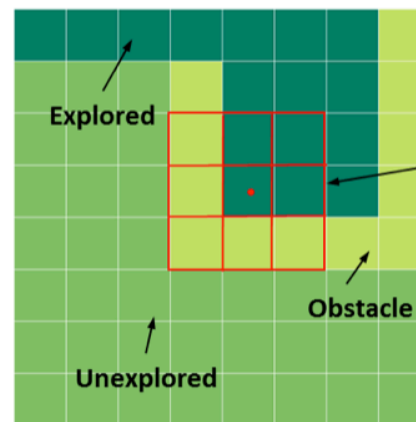
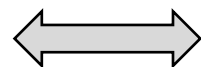
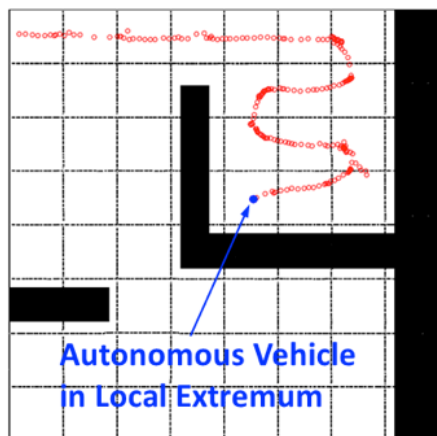
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1

Symbolic encodings at Level 0

E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U
E	F	F	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	O	F	U	U	U	U
E	F	F	F	U	U	U	U
E	U	U	U	U	U	U	U
E	U	U	U	U	U	U	U



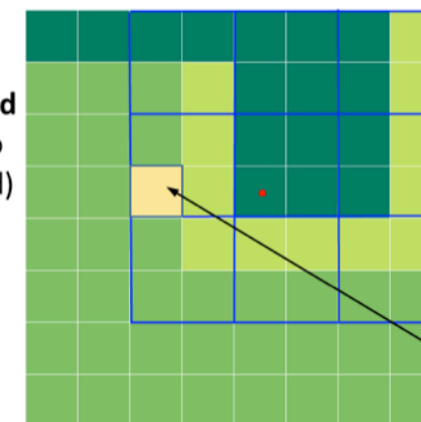
An Example of using MAPS to Prevent the Local Extrema Situation



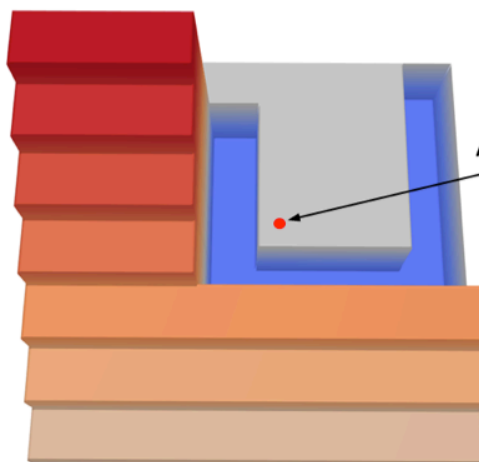
Level 0 Neighborhood
(9 ϵ -cells scanned, no unexplored cell found)

Symbolic Encoding at Level 0

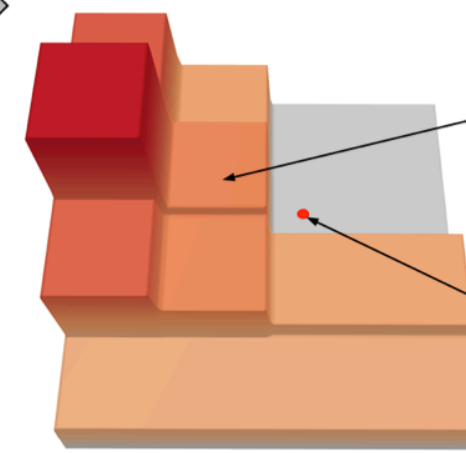
$\ell \leftarrow \ell + 1$



Symbolic Encoding at Level 1



Potential Surface at Level 0



Potential Surface at Level 1

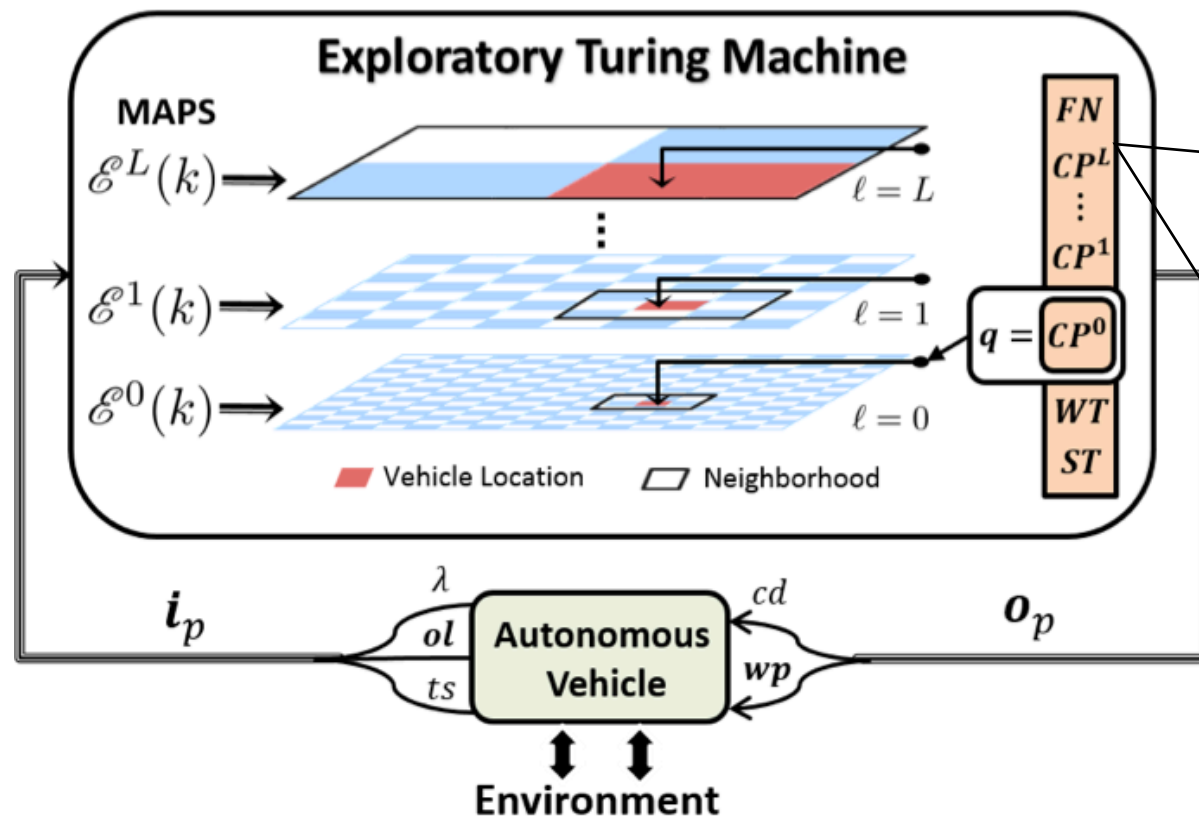


Local Extrema: no unexplored cells are available in the local neighborhood on Level 0.

Low Complexity: even in the worst-case scenario, it only takes $O(|N^0| + L \cdot |N^\ell|)$ to find waypoints, where N^ℓ is the local neighborhood on Level ℓ of MAPS, $\ell = 0, 1, \dots, L$.

ϵ^* : The Supervisory Control Structure

The Exploratory Turing Machine (ETM)



Machine States

- **The Start State (ST):** start the machine and initialize the MAPS with all ϵ -cells as unexplored.
- **The Computing States (CP):**
 - CP^0 : compute waypoint wp using Level 0 of MAPS, and send navigation command cd .
 - CP^1, CP^2, \dots, CP^L : sequentially used to compute wp in case of a *local extremum*.
- **The Waiting State (WT):** wait for the vehicle to complete specific task (e.g., cleaning) in the current cell, until the status ts turns to *complete*
- **The Finished State (FN):** terminate the operation upon complete coverage.

ϵ^* Algorithm

The State Transition Graph

Conditions:
 $\mathcal{A}: wp = \emptyset; \neg\mathcal{A}: wp \neq \emptyset$
 $\mathcal{B}: wp = \lambda; \neg\mathcal{B}: wp \neq \lambda$
 $\mathcal{C}: ts = cm; \neg\mathcal{C}: ts = ic$

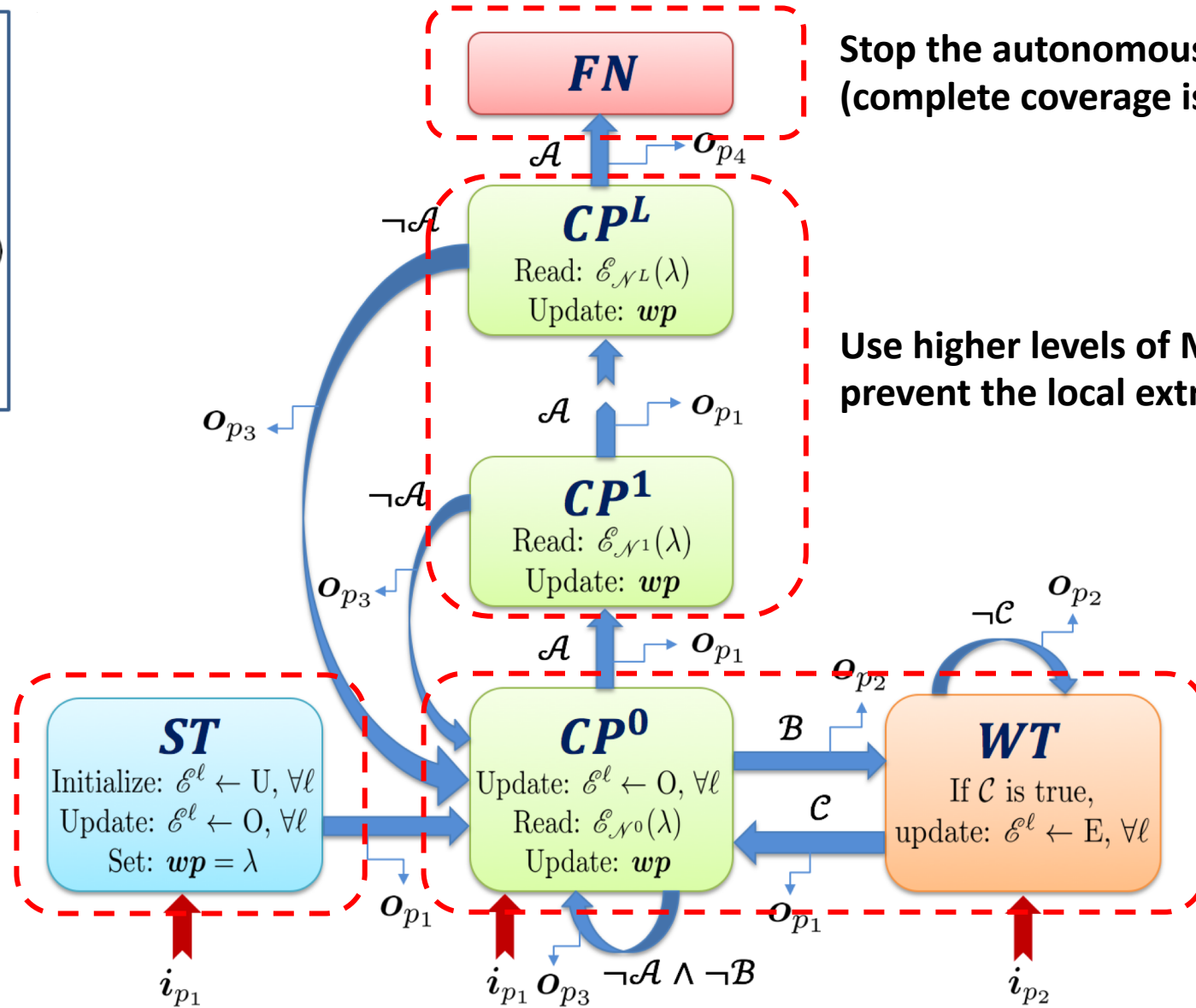
Input Vectors:
 $i_{p1} = (\lambda, ol, -); i_{p2} = (-, -, ts)$

Output Vectors:
 $o_{p1} = (id, -); o_{p2} = (tk, -)$
 $o_{p3} = (mv, wp); o_{p4} = (sp, -)$

Legend:

- ↑ Input Vector
- ↘ Output Vector
- State Transition

System initialization



Stop the autonomous vehicle
(complete coverage is achieved)

Use higher levels of MAPS to
prevent the local extrema problem

Default states for
vehicle navigation
and control

Operation of the ETM: The ST State

When is it reached?

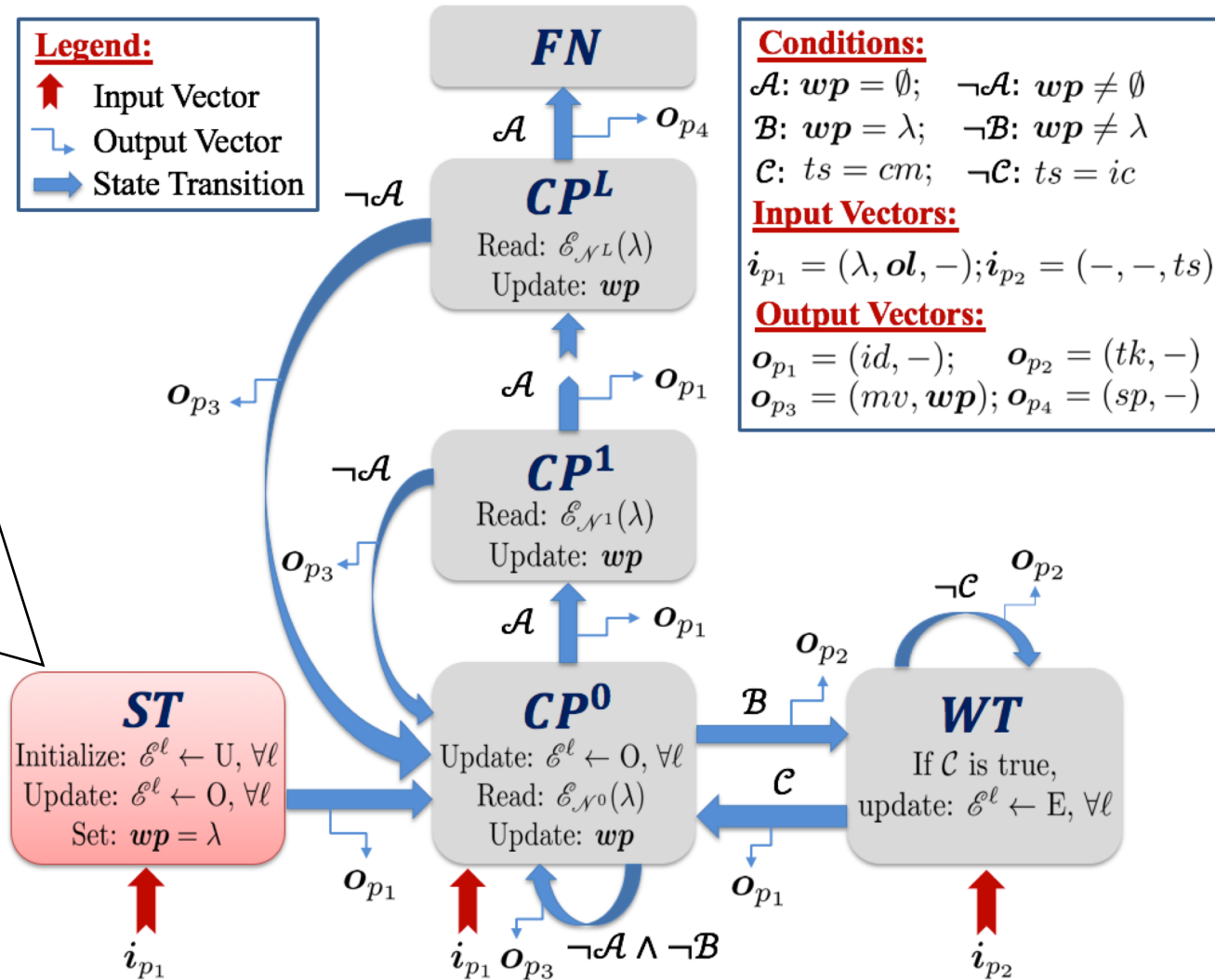
As soon as the autonomous vehicle is turned on.

What does it do?

Initialization of the system.

Operation in the ST State:

- **Initialization:**
 - MAPS $\mathcal{E}^\ell, \forall \ell = 0, 1, \dots, L$:
 - Level 0: initialized with U , i.e. *Unexplored*.
 - Level $1 \leq \ell \leq L$: all coarse cells $\tau_{\alpha^\ell} \in T^\ell$ are assigned potentials by substituting $p_{\alpha^\ell}^U(0) = 1$.
 - Initialize wp as the current cell λ .
- **Input:** the input vector i_{p_1} contains the current vehicle location λ and detected obstacle locations ol
- **Output:** Set the vehicle to idle via output vector o_{p_1} .



Operation of the ETM: The CP^0 State

When is it reached?

Either after system initialization, or when the current cell has just been tasked and needs a new wp .

What does it do?

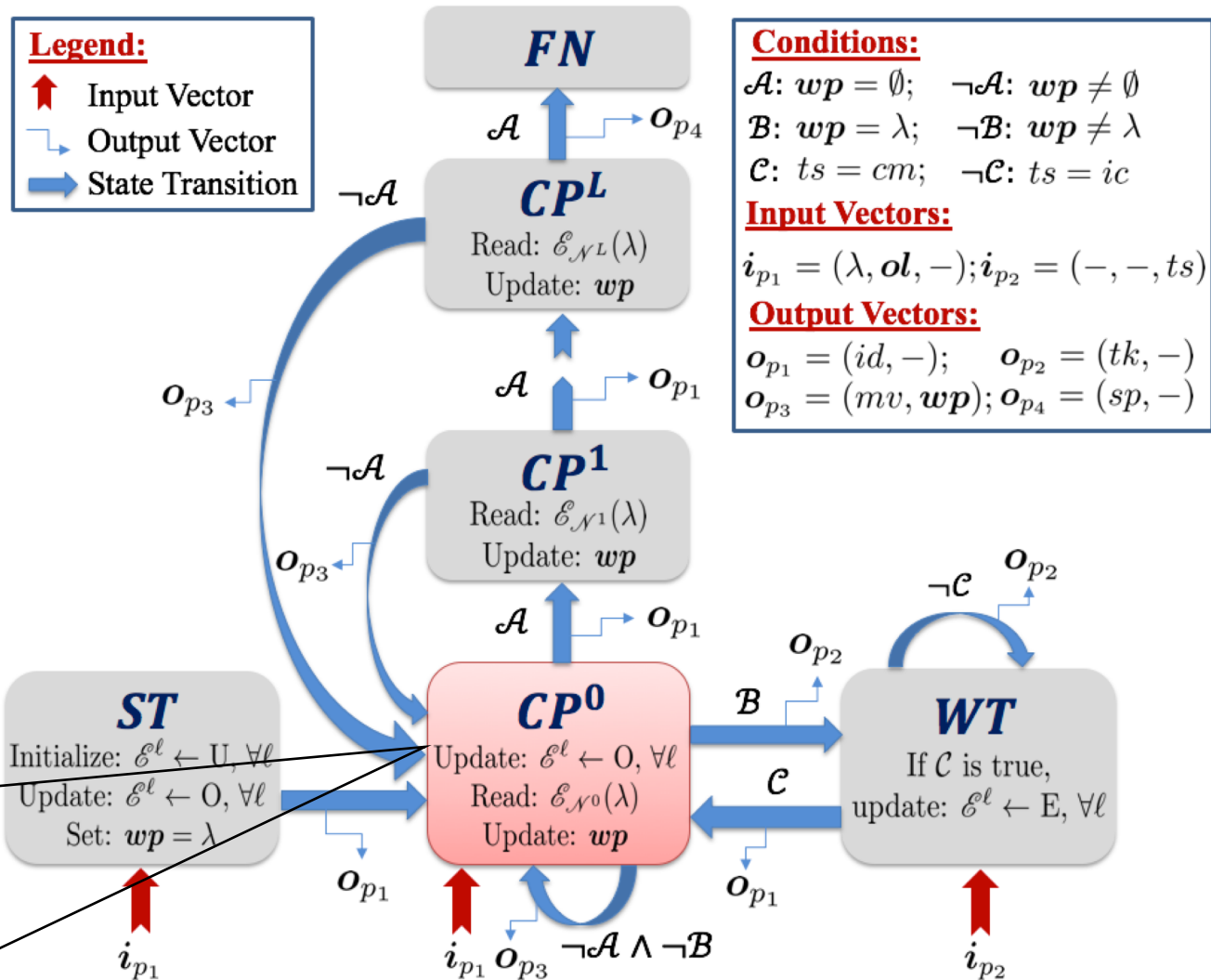
Default state to compute for wp on Level 0 of MAPS.

Operation in the CP^0 State:

- **Input:** the input vector i_{p_1} contains vehicle location λ , obstacle locations ol ; they are used to update potential surfaces $\mathcal{E}^\ell, \forall \ell$.
- **Compute wp :** the directly reachable neighbor cell with the highest positive potential in the neighborhood N^0 :

$$wp(k) = \text{arcm}ax_{\alpha^0 \in N^0} \mathcal{E}_{\alpha^0}$$

- **Output:** If wp is found, send vector o_{p_3} to move vehicle to wp ; and upon reaching, send vector o_{p_2} to start tasking.
 - If wp not found, switch to state CP^1



Operation of the ETM: The *WT* State

When is it reached?

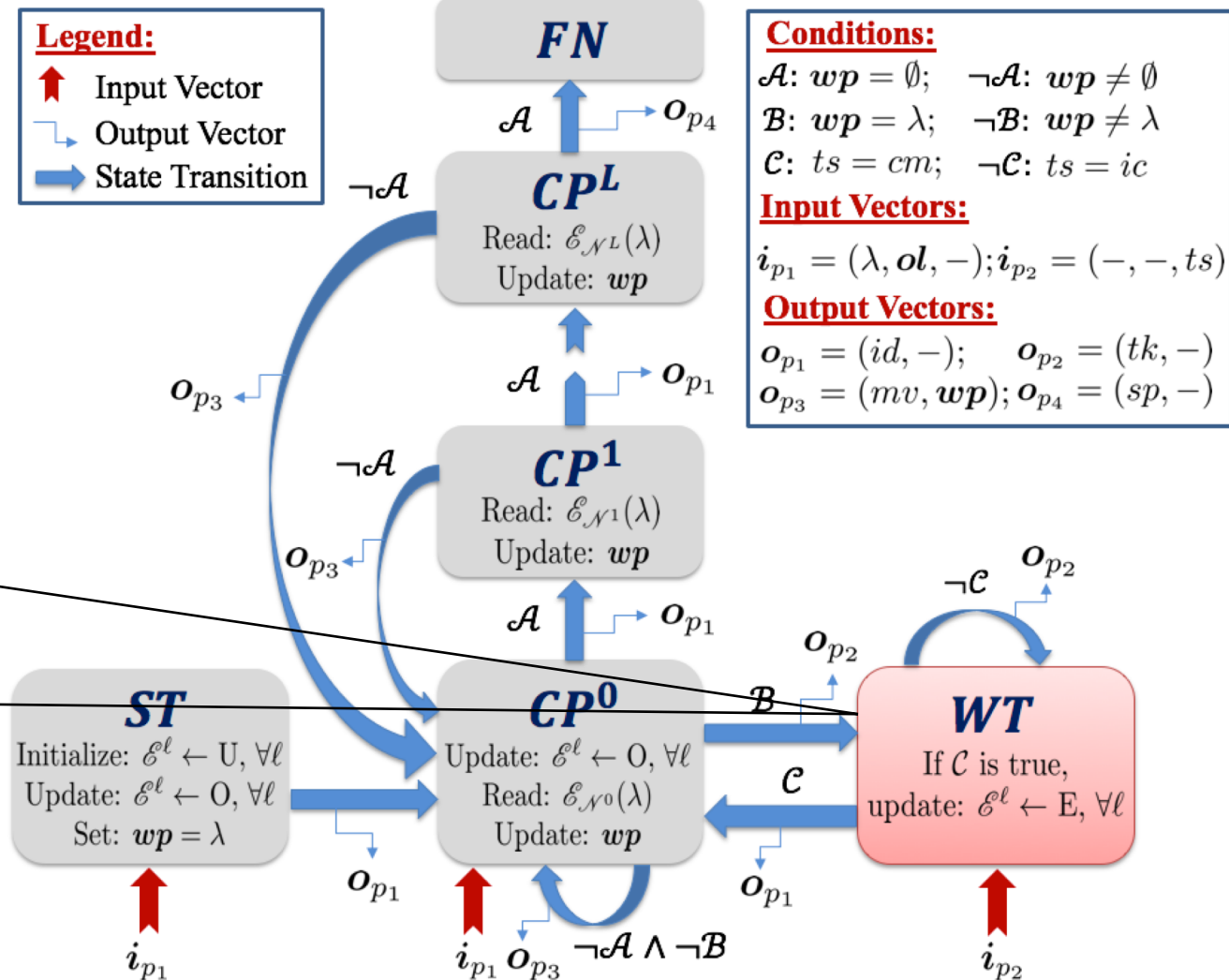
When the autonomous vehicle reaches the computed *wp*.

What does it do?

Command the vehicle to perform tasking (e.g., cleaning) in the current cell.

Operation in the *WT* State

- **Input:** the input vector i_{p_2} contains task status ts ; if it is *complete*, update current cell λ as *E*, i.e., *Explored*; then update potential surfaces $\mathcal{E}^\ell, \forall \ell$.
- **Output:**
 - If task status ts is *complete*, go to state CP^0 to compute for the next *wp*
 - Otherwise, keep on waiting.



Operation of the ETM: The $CP^1, CP^2, \dots CP^L$ States

When are they reached?

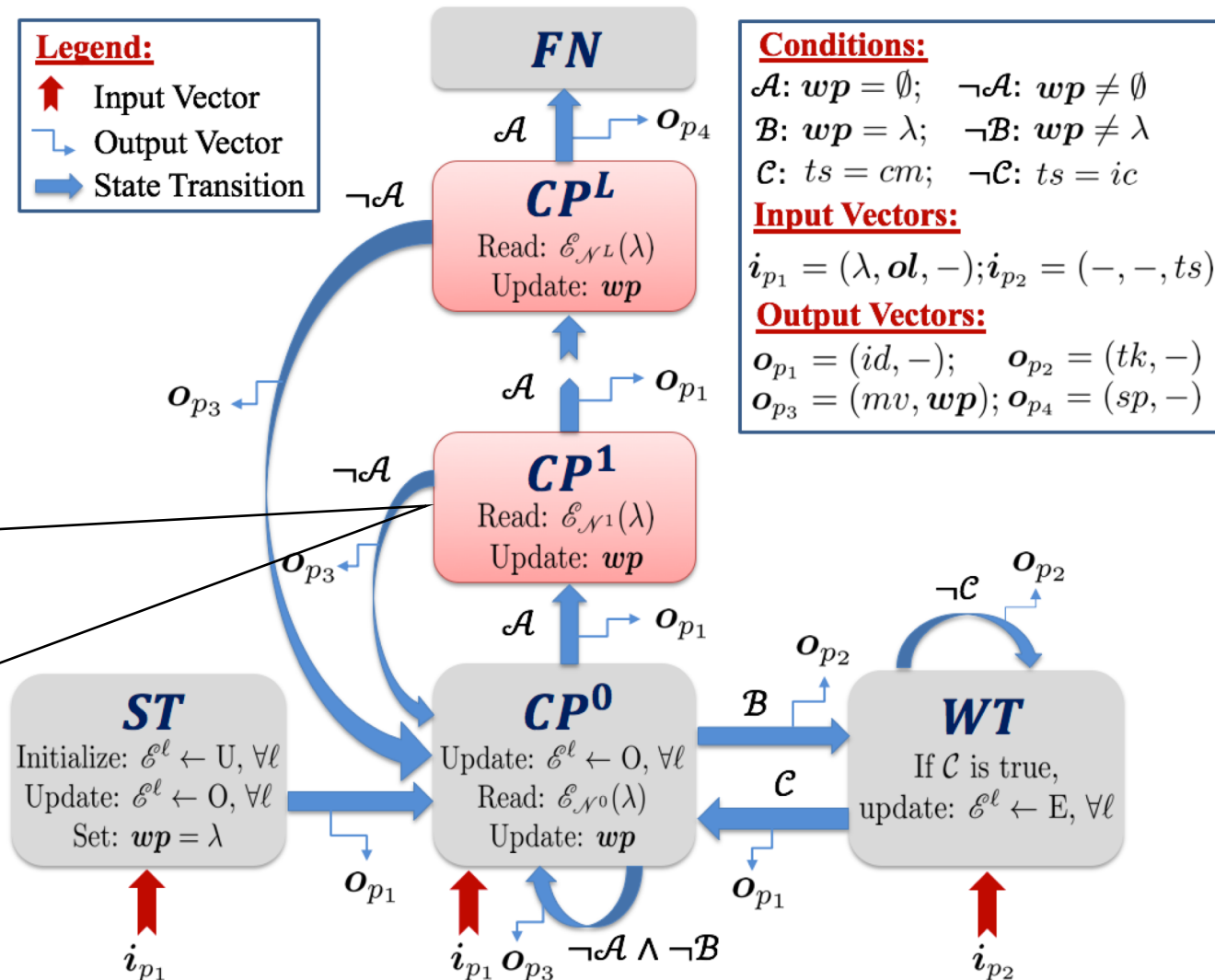
When waypoint wp cannot be found in CP^0 state.

What does it do?

Sequentially switches to higher levels of MAPS, until wp can be found at some Level $\ell \leq L$.

Operation in the $CP^\ell, \ell = 1, 2, \dots L$ States

- Compute wp**
 - First, read the potentials in the local neighborhood on Level 1 (i.e., $\mathcal{E}_{N^1}(\lambda)$).
 - If $\exists \tau_{\alpha^1} \in N^1(\lambda)$ with positive potential, then wp is set as an unexplored ϵ -cell in τ_{α^1} .
 - Otherwise, go to CP^2 state and repeat above.
- Output:** If wp is found, sends output o_{p_3} to move vehicle to wp ; otherwise, sends o_{p_1} to set vehicle idle.



When is it reached?

When wp cannot be found in CP^L state.

What does it do?

Terminate operation since no unexplored cells are left.

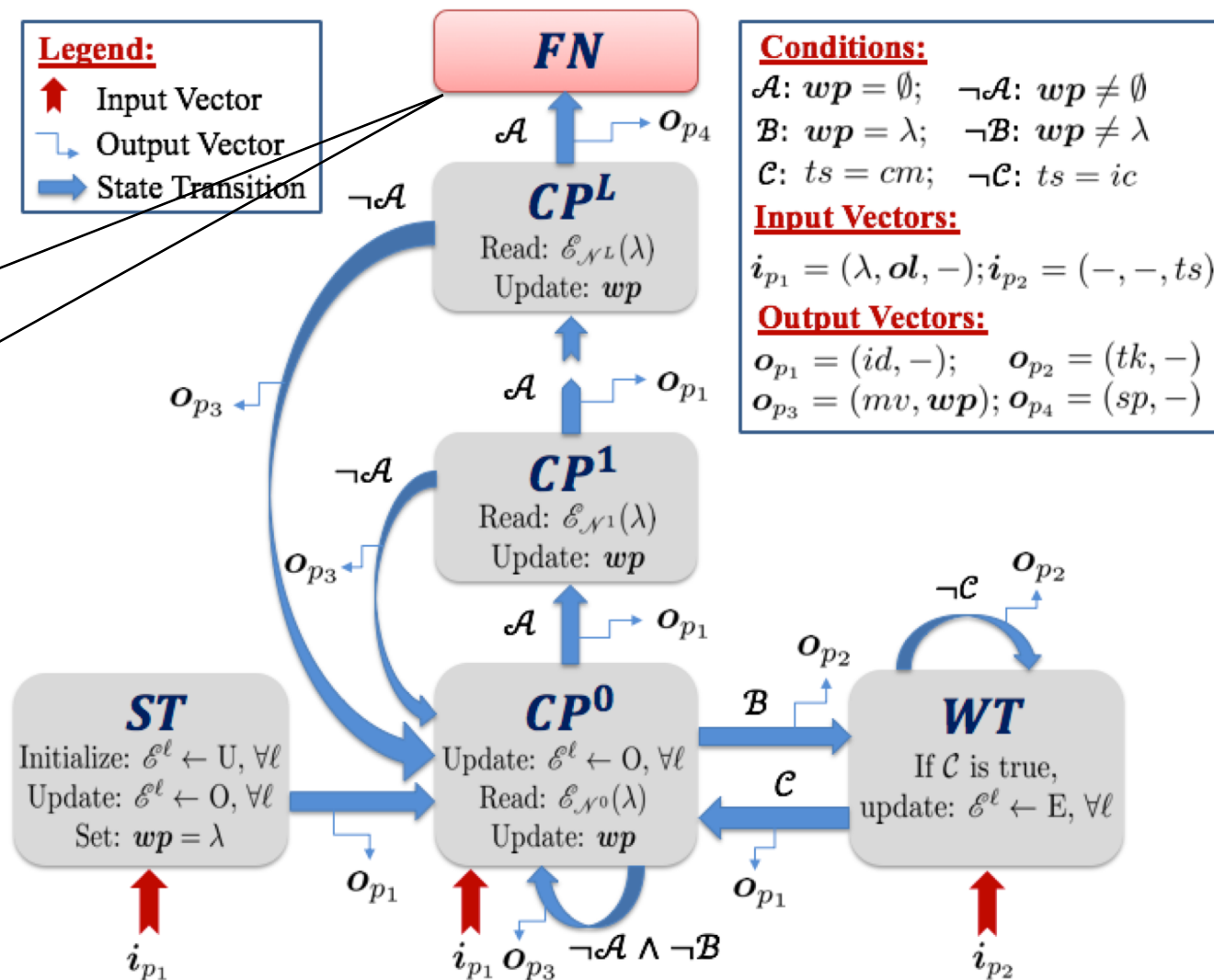
Operation in the FN State

- If $wp = \emptyset$ at Level L , it implies the ETM cannot find unexplored ϵ -cells at the highest Level L , then it reaches the FN state and the machinery is terminated.

Theorem: The ETM halts in finite time^[1].

Corollary 1: Each allowed ϵ -cell is tasked only once^[1].

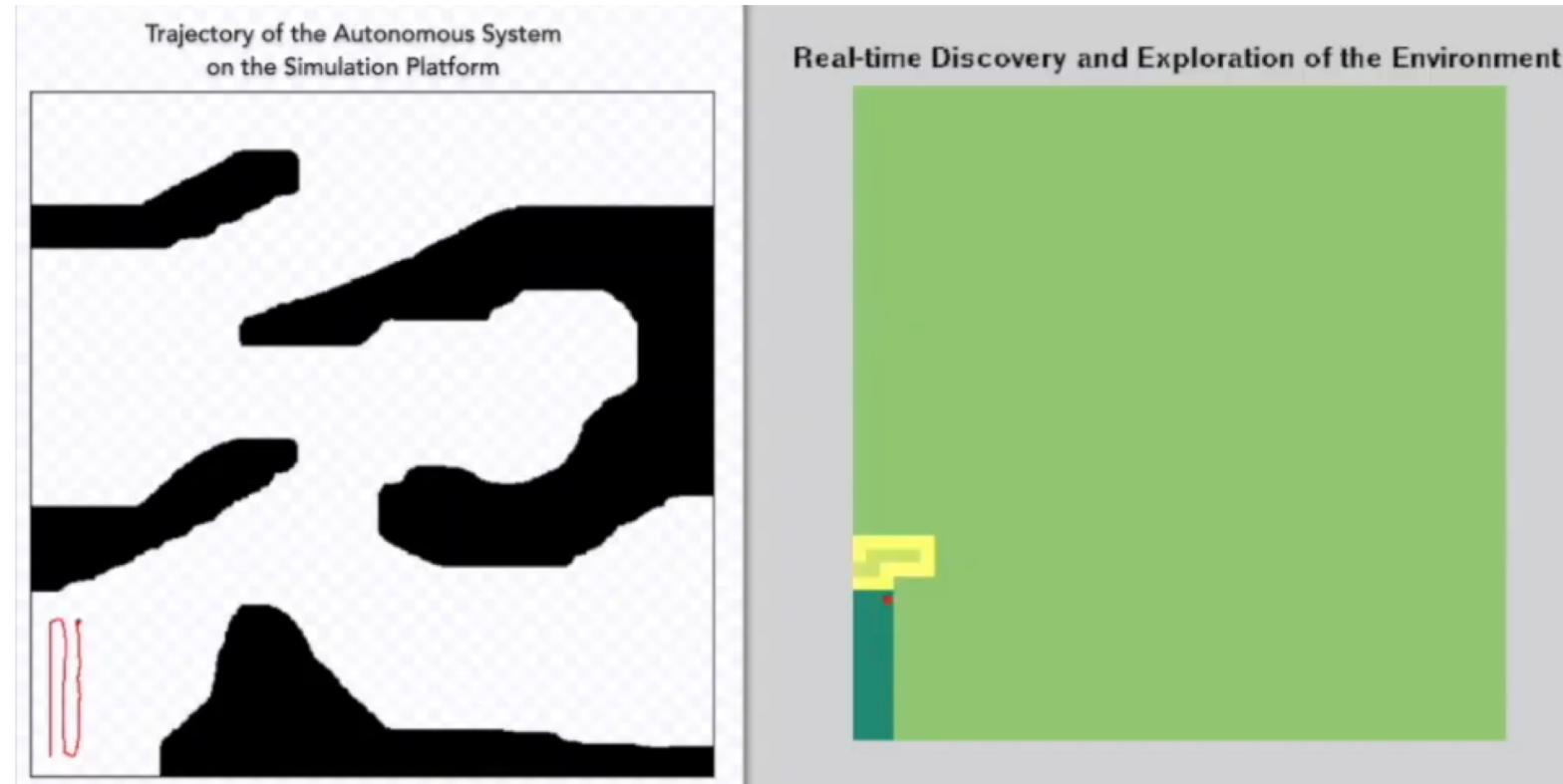
Corollary 2: ϵ -coverage is achieved upon halting^[1].



Validations on the Player/Stage Robotic Simulator

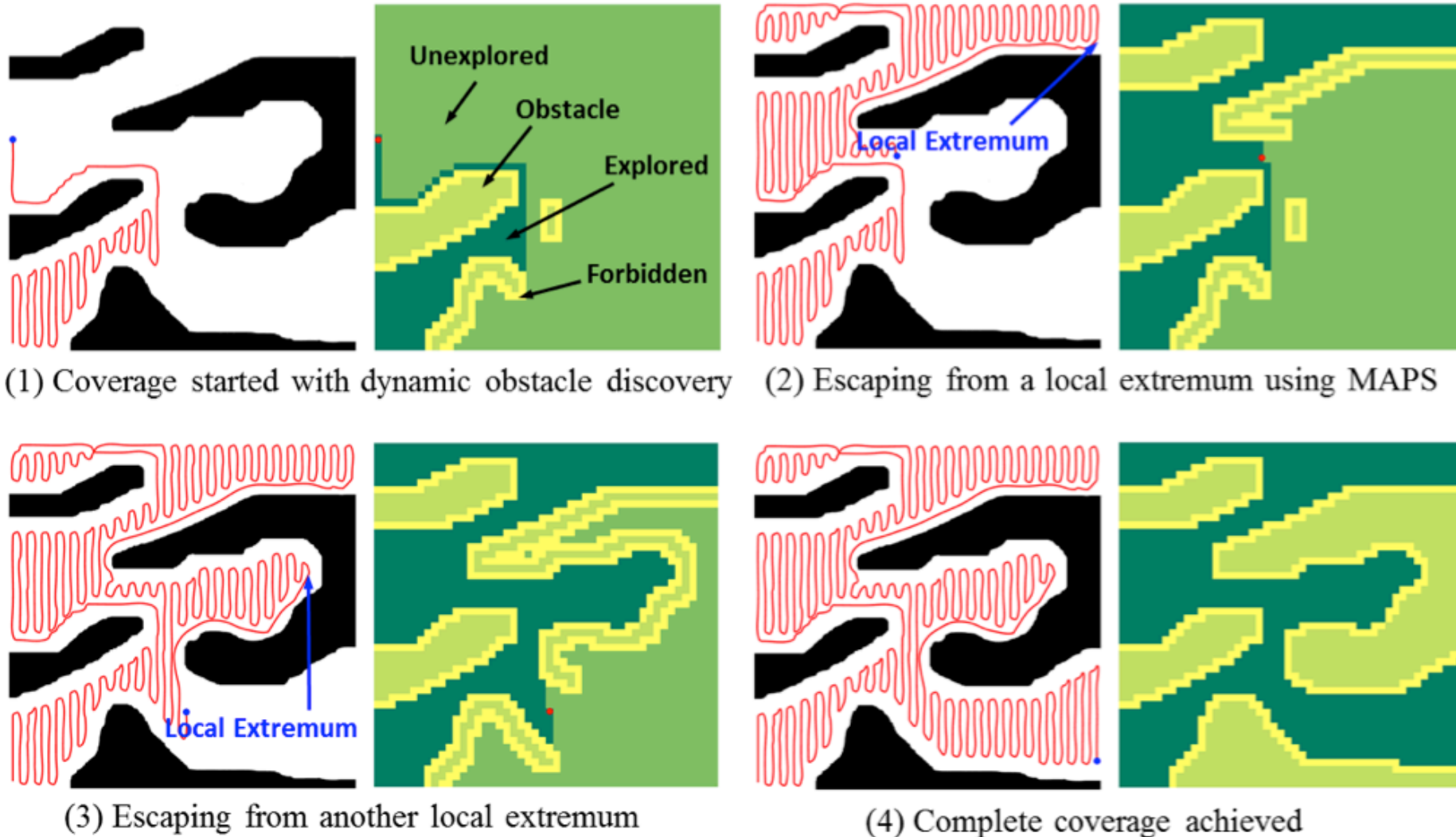
❖ Simulation Setup

- **Autonomous Vehicle:** a Pioneer AT2 of dimensions $0.44\text{m} \times 0.38\text{m} \times 0.22\text{m}$ was used with kinematic constraints of:
 - Top speed: 0.5m/s
 - Maximum acceleration: 0.5m/s^2
 - Minimum turning radius: 0.04m
- **Sensing systems**
 - Laser: detection range of 4m
- **Search Area:** the search area is of size $50\text{m} \times 50\text{m}$, which is partitioned into a 50×50 tiling consisting of ϵ -cells of size $1\text{m} \times 1\text{m}$. This results in MAPS with $L = 5$ levels.



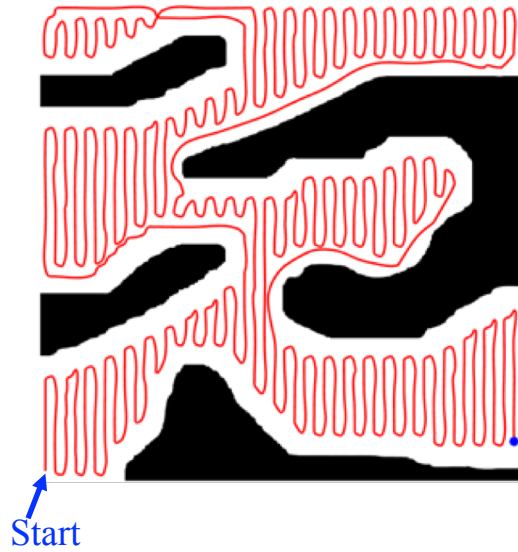
Scenario 1: Coverage Trajectories and Symbolic Encodings of the Environment

- ϵ^* incrementally builds the environment map, and complete coverage is achieved.

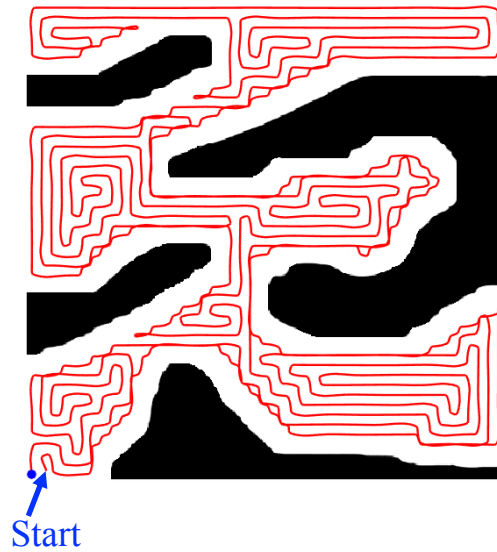


Scenario 1: Comparison with Alternative Methods

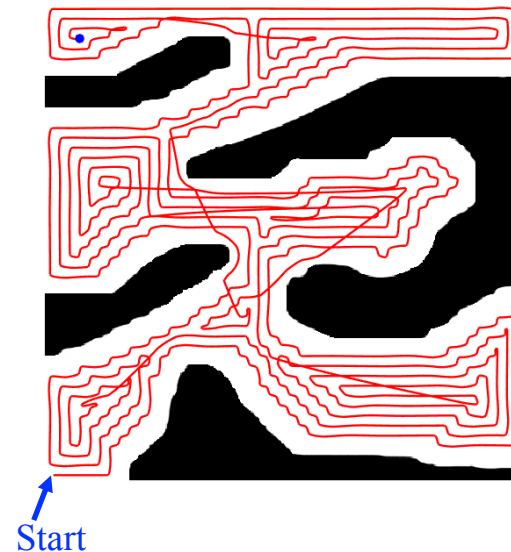
(a) ϵ^* Algorithm



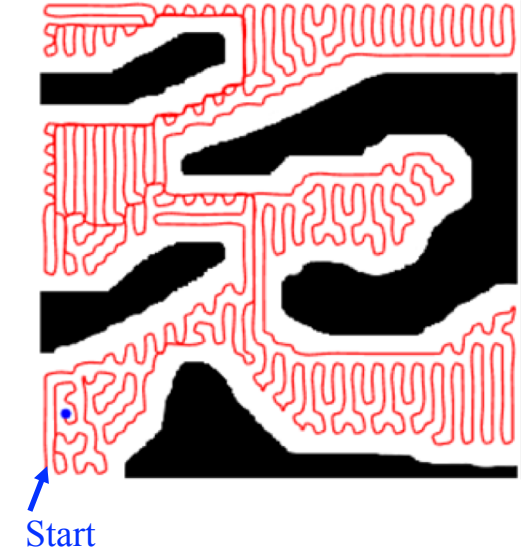
(b) Spanning Tree Coverage



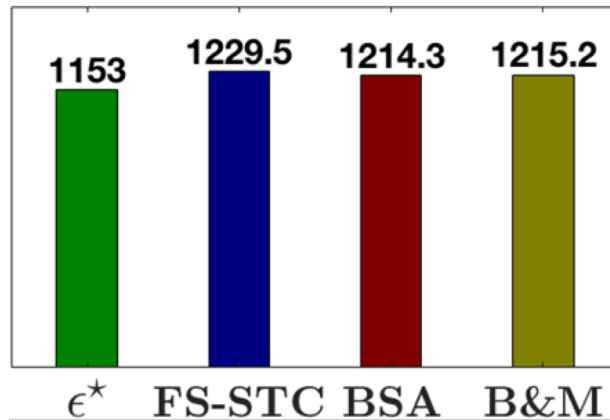
(c) Backtracking Spiral Algorithm



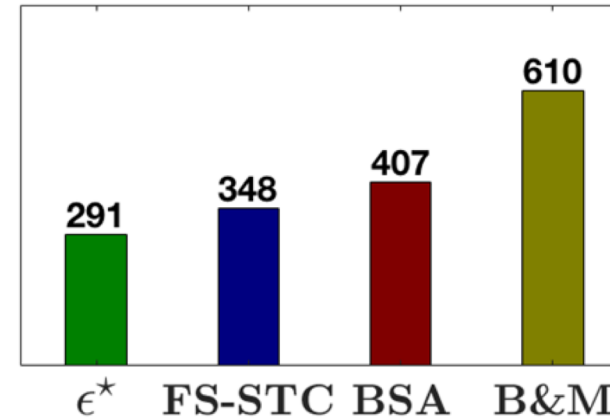
(d) Brick & Mortar



Trajectory length



Number of turns



Scenario 2: Adaptive Sweep Direction in Known Sub-regions

❖ User-controllable Sweep Direction

- If provided (partial) environment knowledge in sub-regions, ϵ^* can adapt the sweep direction to further reduce the number of turns.
- In Scenario 2 below, the layouts of all rooms are assumed **known**, but the inside obstacles are **unknown**.
- Then, the field B was designed in a manner such that the AV sweeps the top left room horizontally while the other two rooms vertically

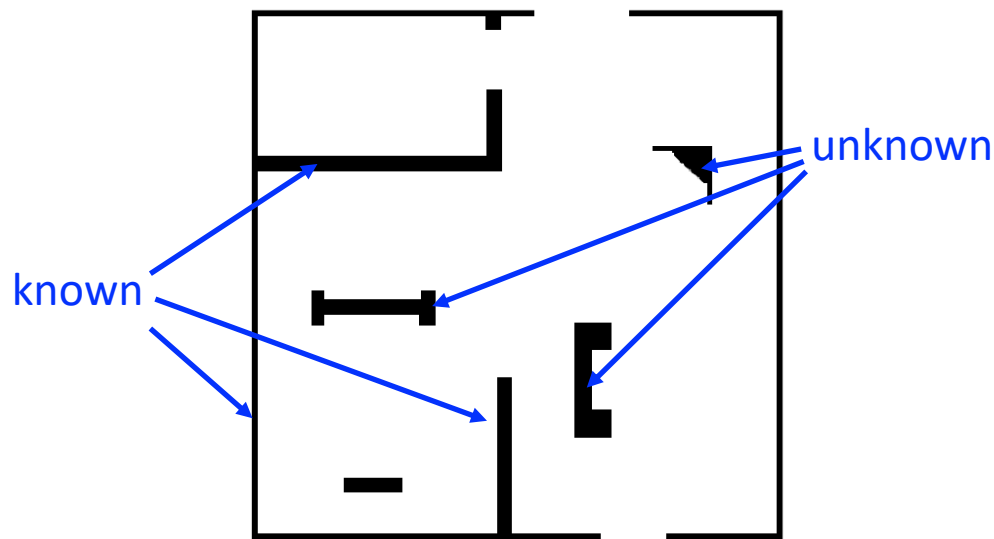


Fig. Scenario 2

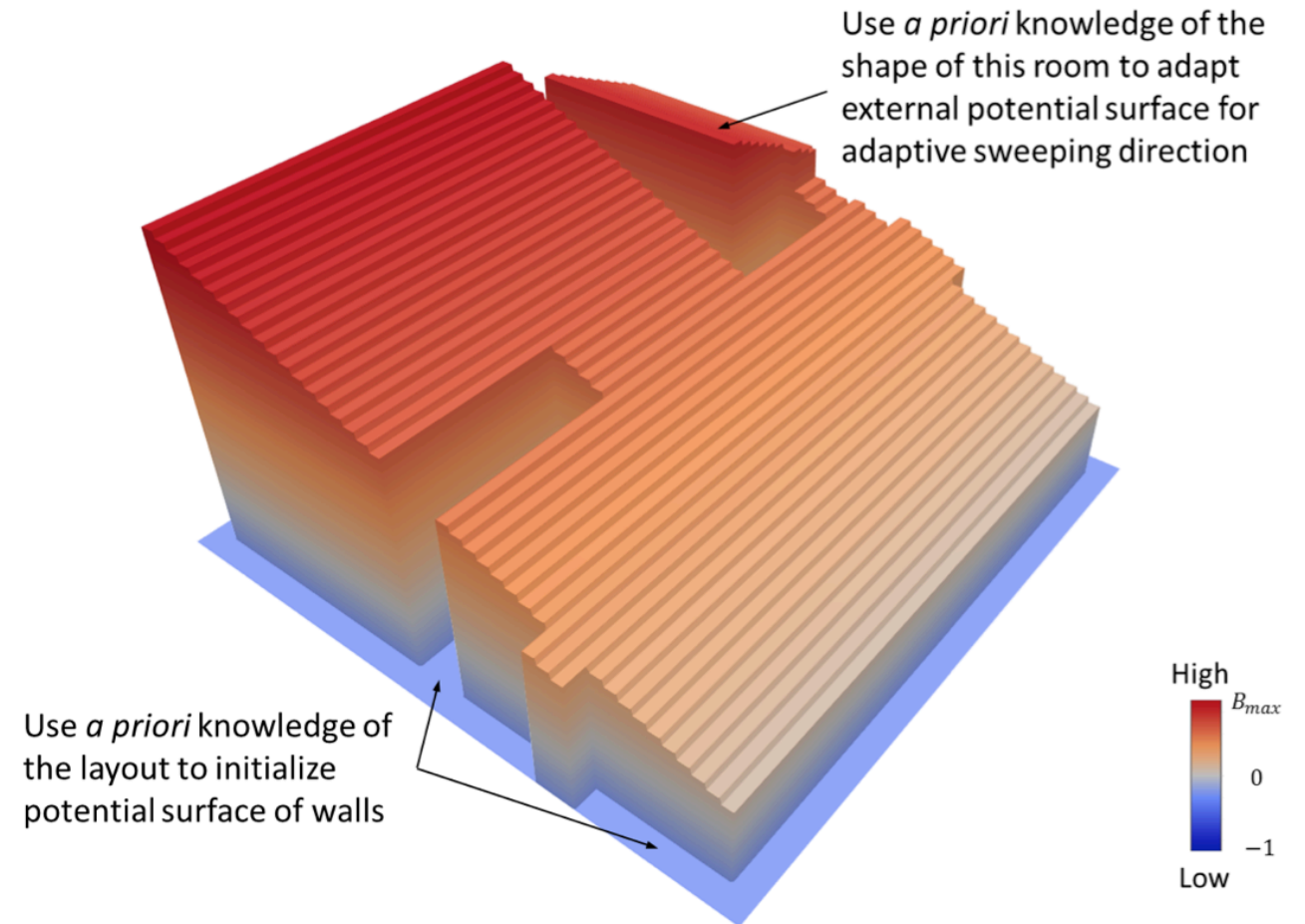
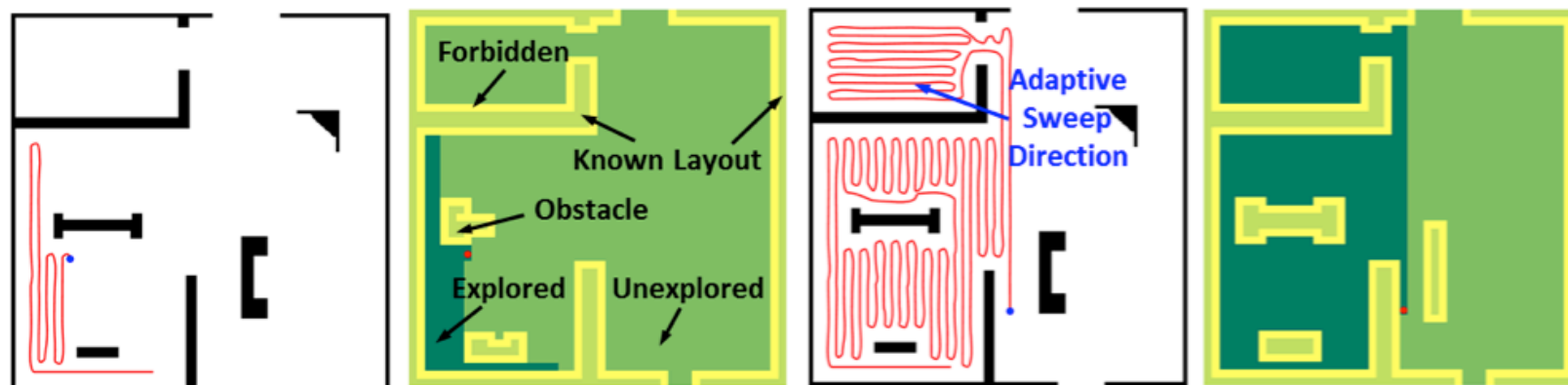


Fig. Exogeneous potential field B in Scenario 2

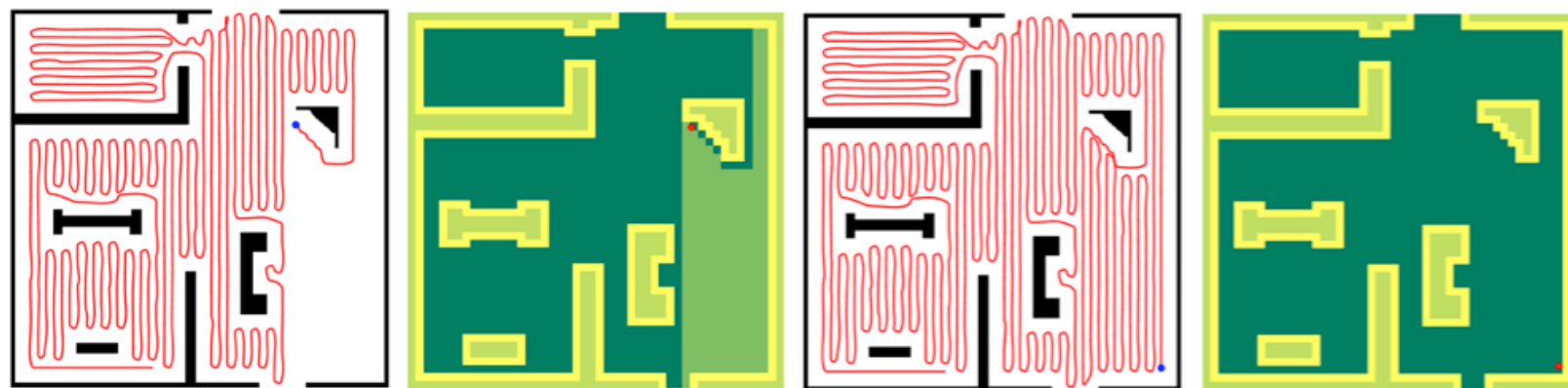
Scenario 2: Adaptive Sweep Direction in Known Sub-regions

- User-controllable Sweep Direction:** If provided (partial) environment knowledge in sub-regions, the sweep direction can be adapted to further reduce the number of turns. This is done by altering the exogeneous potential field B .

Scenario 2: Coverage trajectory of ϵ^* in an apartment scenario



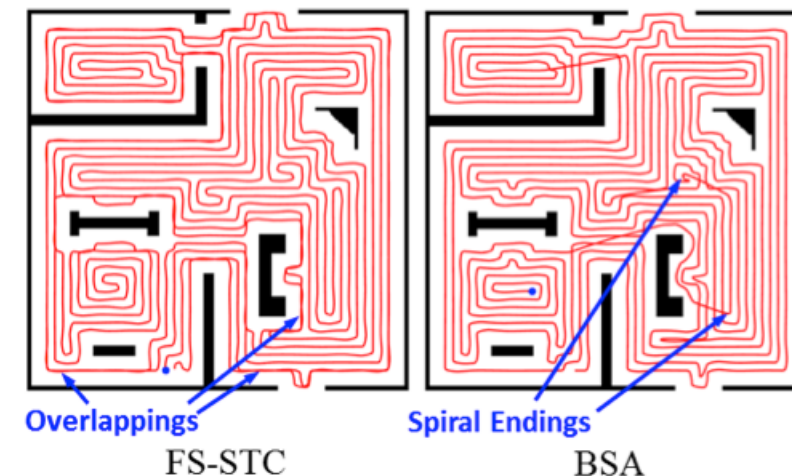
(1) Coverage started with dynamic obstacle discovery (2) Adaptive sweeping if layout is *a priori* known



(3) Adapt to the shape of obstacle

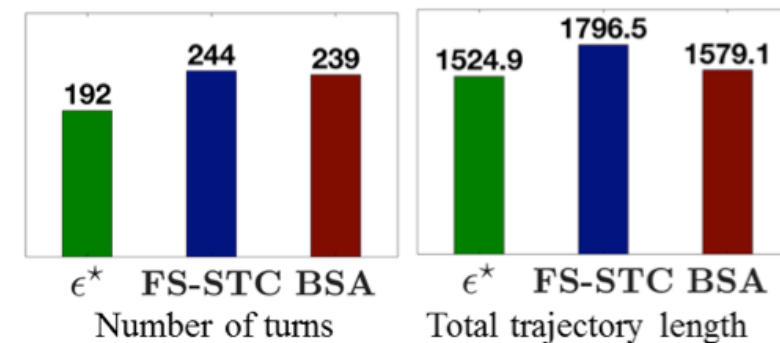
(4) Complete coverage achieved

Trajectories of alternative methods



FS-STC

BSA



Coverage Performance under Uncertainties

❖ Coverage Ratio r_c :

$$r_c = \frac{U_k \tau(k)}{\mathcal{R}(T^a)}$$

❖ Sources of Uncertainties:

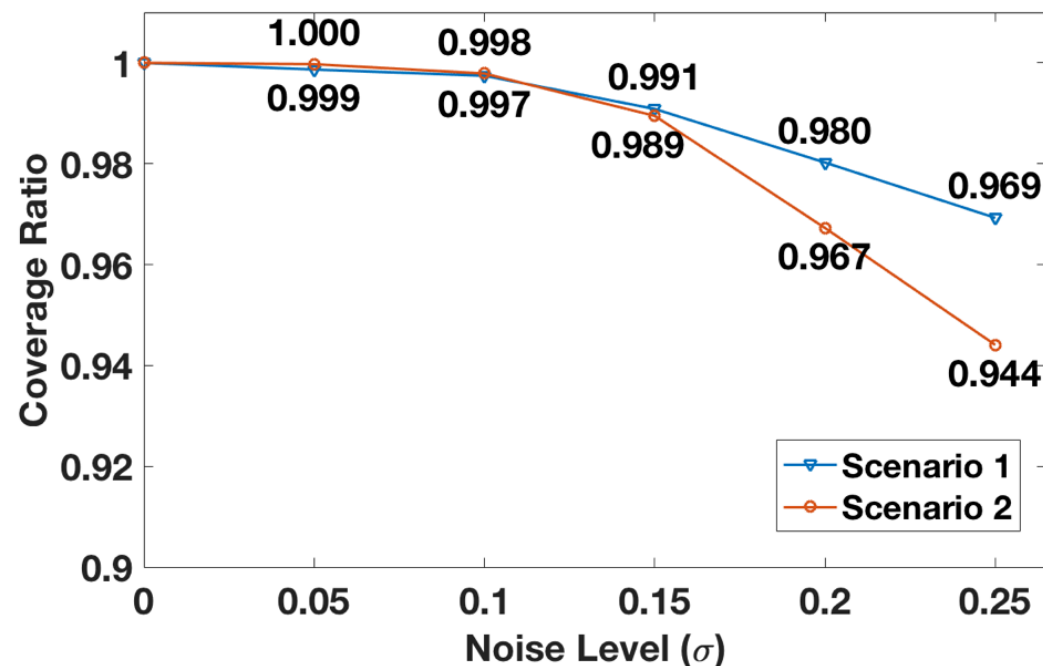
- **Localization System:**
 - Outdoor: Real-time Kinematic (RTK) GPS can achieve an accuracy of 0.05m~0.5m^[1].
 - Indoor: Hagisonic StarGazer indoor localization system provides precision of 2cm.
- **Compass:** a modestly priced compass provides an accuracy of 1°^[1].
- **Laser Measurements:** a laser sensor typically admits an error of 1% of its operation range.

❖ Monte Carlo Simulations:

The sensor noise are simulated as Additive White Gaussian Noise (AWGN), with:

- **Localization System:** $\sigma = 0.05\text{m}, 0.10\text{m}, \dots 0.25\text{m}$
- **Compass:** $\sigma_{compass} = 0.5^\circ$
- **Laser Measurements:** $\sigma_{laser} = 1.5\text{cm}$

Coverage ratio vs. noise for ten Monte Carlo runs



[1] L. Paull, S. Saeedi, M. Seto, and H. Li, "Auv navigation and localization: A review," IEEE Journal of Oceanic Engineering, vol. 39, no. 1, pp. 131–149, 2014.

[2] J. Palacin, J. A. Salse, I. Valganon, and X. Clua, "Building a mobile robot for a floor- cleaning operation in domestic environments," IEEE Transactions on Instrumentation and Measurement, vol. 53, no. 5, pp. 1418–1424, 2004.

Choosing a Proper Sized ϵ

❖ Selection of the Size of ϵ :

- Should be big enough to contain the autonomous vehicle, and small enough for the tasking sensor to be able to cover it.
- Within these two bounds, the choice of ϵ depends on the following factors:
 - Smaller ϵ : provides a better approximation of the search area and its obstacles.
 - Larger ϵ : reduces the computational complexity by requiring less number of ϵ -cells to cover the area and it also provides improved robustness to uncertainties for localization within a cell.

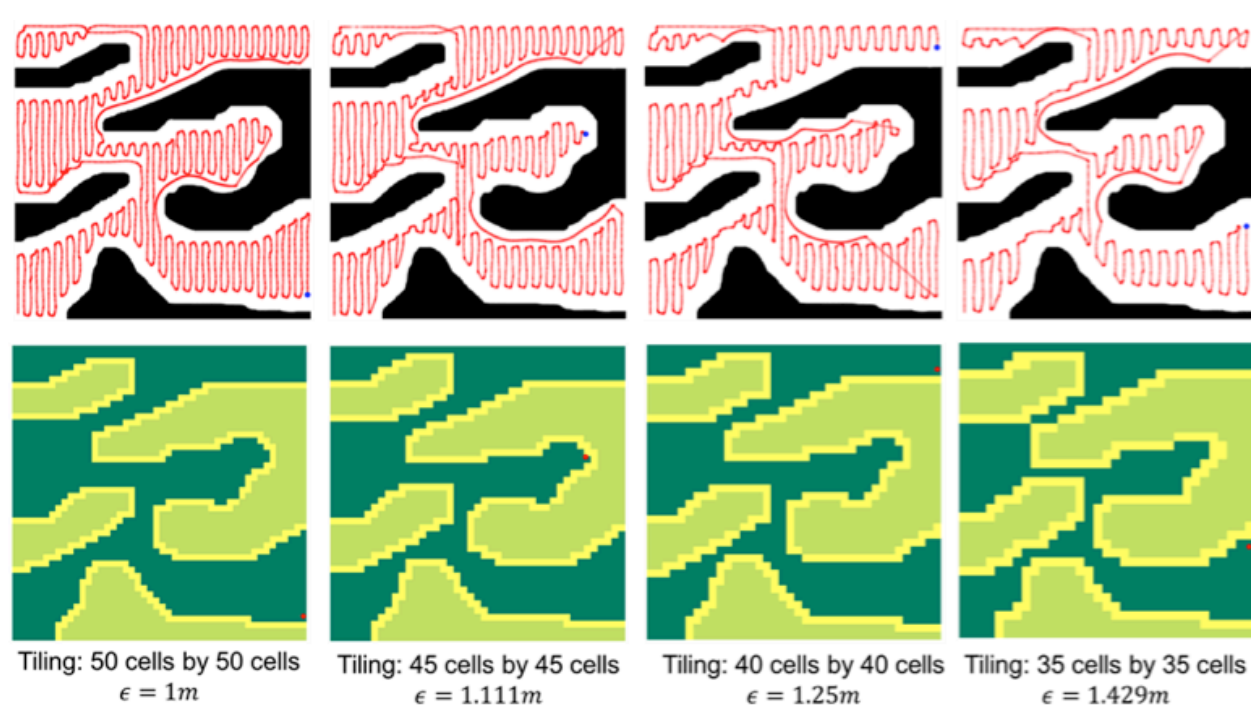


Figure 1. Scenario 1: coverage trajectories for varying size of ϵ

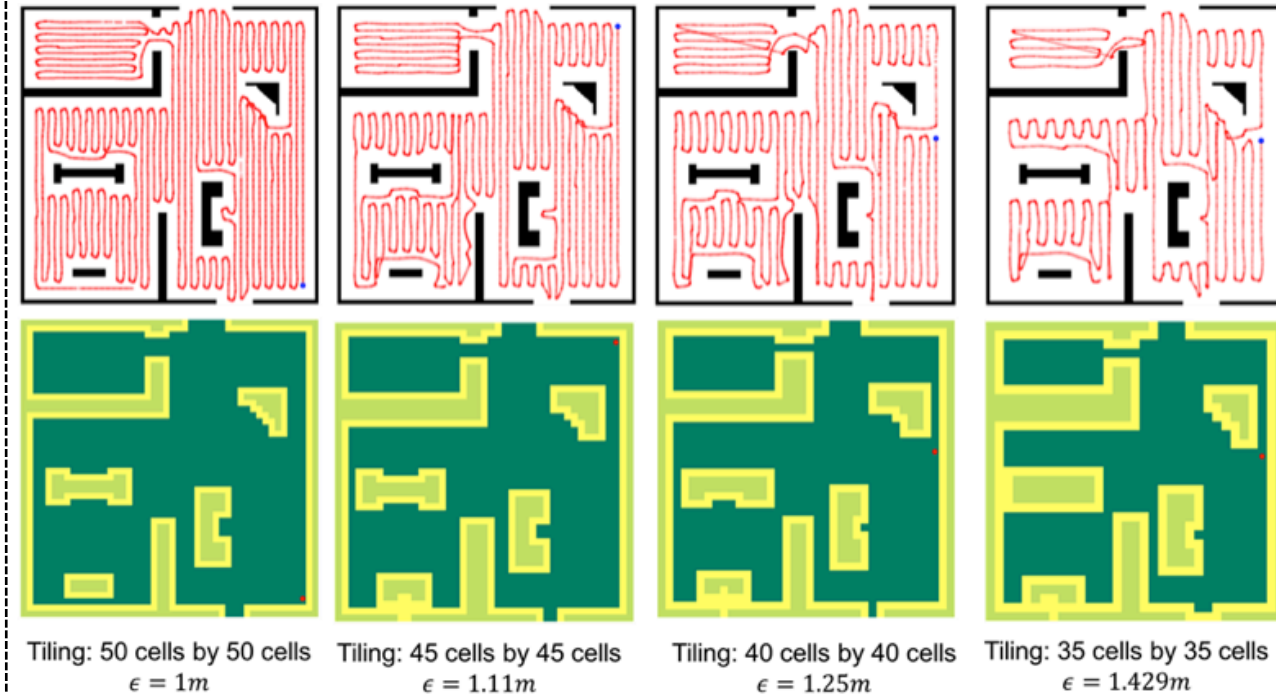
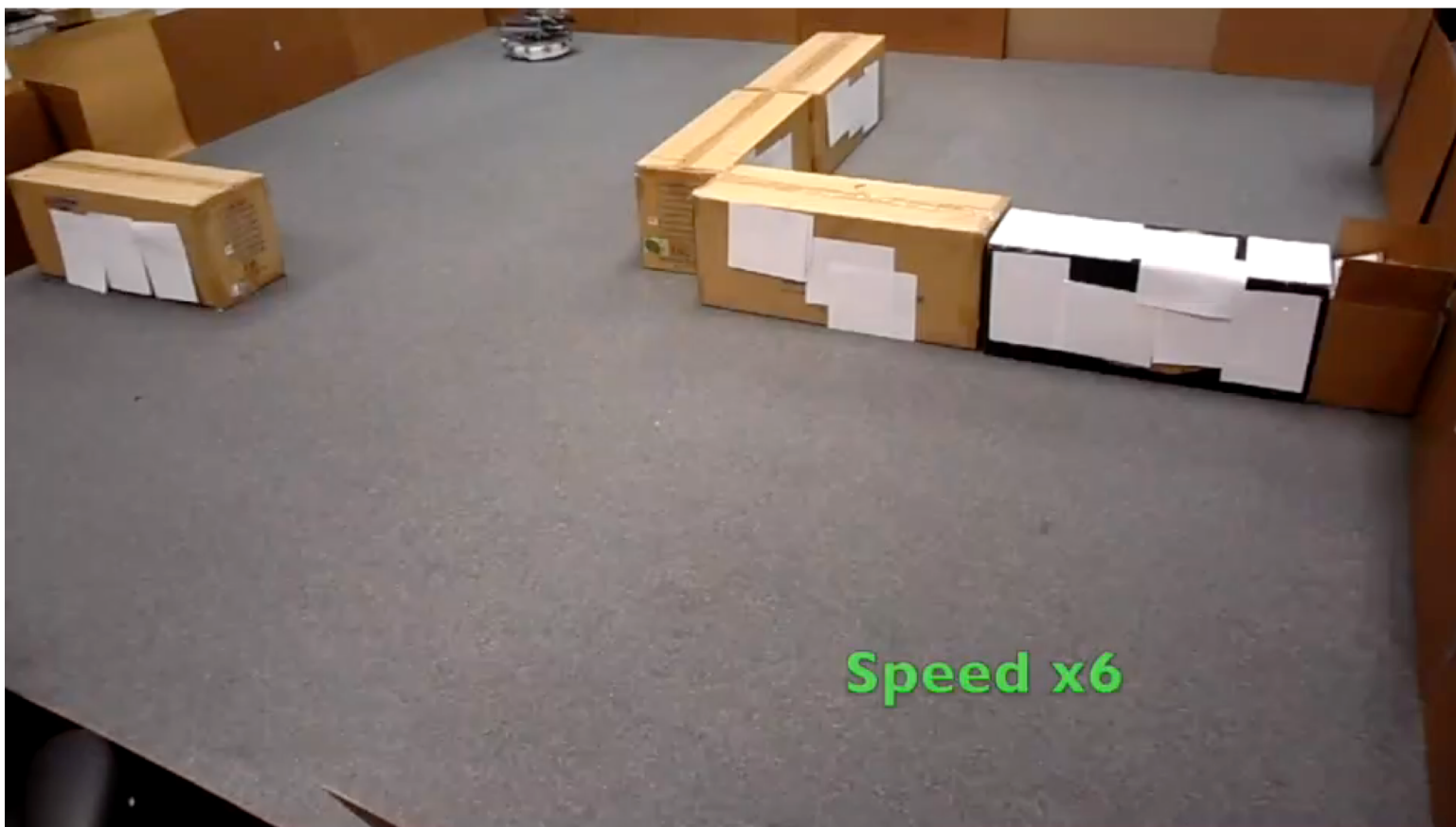


Figure 2. Scenario 2: coverage trajectories for varying size of ϵ

The Autonomous Ground Vehicle (AGV)

- ❖ ϵ^* algorithm was validated in real laboratory-scale experiments to address real-life uncertainties in sensing and vehicle control
- ❖ iRobot Create was used as the AGV, which is *programmable* and *controllable* using feedbacks from popular sensing devices



An AGV integrated with multiple sensing devices

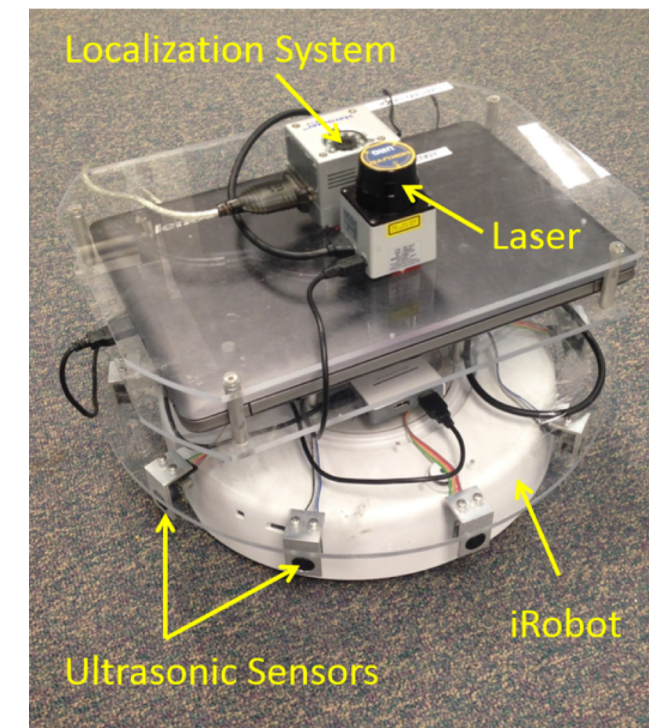


Table. Specifics of the on-board sensing systems

	Localization	Laser	Ultrasonic
Model	StarGazer	URG-04LX	XL-MaxSonar-EZ
Range	–	0.02m ~ 5.6m, 240°	0.2m ~ 7.65m
Resolution	1cm, 1°	1mm, 0.36°	1cm
Accuracy	2cm, 1°	±1% of Measurement	–